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# Stochastic DEA with Using of Skew-Normal Distribution in Error Structure

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**Abstract**

The stochastic data envelopment analysis (SDEA) was developed considering the value of inputs and outputs as random variables. Therefore, statistical distributions play an important role in this regard. The skew-normal (SN) distribution is a family of probability density functions that is frequently used in practical situations. In this paper, we assume that the input and output variables are skew-normally distributed. With introducing asymmetric error structure for random variables of SN distribution, a stochastic BCC model is provided. The proposed model includes BCC model assuming a normal distribution of data as well. Finally, the proposed model is used in a numerical example.

***Keywords:*** *Data Envelopment Analysis, Efficiency, Skew-normal Distribution, Error Structur*

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## 1. Introduction

Data envelopment analysis (DEA) is a technique which is based on mathematical programming for measuring the relative efficiency of a set of decision making units (DMUs). Founder of Non-parametric methods in calculating of efficiency and performance evaluation of DMUs economist named was Farrell [9] which in a paper presented methods of measuring the efficiency of based on economic theory. Charnes et al. [5] in 1978 using mathematical programming, Farrell's non-parametric method for a system with multiple input and output extend and it was named the CCR model. The proposed model is based on the performance's constant returns to scale. The seminal work of Banker et al. [3] developed a variable return to scale version of the CCR model and introduced what is now known as BCC model.

Given the use of stochastic data in practical problems, Thore [18], Li [15], Sengupta [17], Huang and Li [11], Cooper et al [6, 7, 8], Khodabakhshi [12], khodabakhshi and Asgharian [13], Khodabakhshi et al.[14] and Hosseinzadeh-Lotfi et al. [10] proposed DEA models with stochastic data. Different models provided in DEA by the researchers, based on the assumption of normality of the data proposed. The normal distribution is a continuous distribution of statistics is of particular importance. This distribution has many of the properties, which enhances its use. Almost, all of the previous works in stochastic DEA (SDEA) have been used the stochastic data when the inputs and outputs having normal distribution which it is symmetric distribution. But in practical problems, the distributions of variables are often asymmetric and normal distribution can no longer be used for analysis. In such cases some transformations may be used to make the distribution of data closer to normal distribution so that the data can then be analyzed. However, this transformation has its own problems and drawback such as biasedness of estimators. Due to this issue, using asymmetric distributions which have the same characteristics as normal distribution, have received some attention in the literature.

Skew-normal (SN) distribution is one of the most important statistics distributions which are first proposed by Azzalini [1]. The SN distribution is an asymmetric distribution which has similar properties to a normal distribution and it can be extended to the normal distributions. This distribution has one shape parameter which if this quantity is equal to zero; normal distribution can be achieved. Therefore, the normal distribution is a special case of SN distribution. Behzadi and Mirbolouki [4] introduced the symmetric error structure and proposed a linear form of stochastic CCR model. In this paper with the introduction of asymmetric error structure for variables of inputs and outputs with SN distribution, a stochastic BCC mode for measuring the efficiency of DMUs are proposed.

This paper proceeds as follows: In section 2, some basic concepts in statistics and deterministic BCC model are discussed. The asymmetric error structure and deterministic BCC model with SN distribution is presented in Section 3. Section 4 contains a numerical example using the proposed model. Section 5 includes discussion and concluding remarks.

## 2. Preliminaries

In this section, we recall some of the basic concepts of statistics and results which will be use through the article.

### 2.1. Skew-Normal Distribution

Distribution of SN, which was introduced by Azzalini [1], is defined as follows:

***Definition 2.1*** A random variable Z is called to have standard SN distribution, denoted by $Z \sim SN(\delta)$, if its probability density function (PDF) is given as follows:

$$f_Z(z) = 2\phi(z)\Phi(\delta z) \quad (1)$$

Where $\phi$ and $\Phi$ are the PDF and cumulative density function (CDF) of the standard normal distribution respectively. The shape parameter $\delta$ is called “skewnees control parameter”. The PDF (1) for $\delta$s different is drawn in Figure 1.

The CDF of standard SN distribution is as follows:

$$F_Z(z,\delta) = \Psi(z,\delta) = \Phi(z) - 2T(z,\delta), \quad (2)$$

Where $T(h,g)$ is the Owen’s T function, tabulated in Owen [16] and that can the deffined by

$$T(h,g) = \frac{1}{2\pi}\int_0^g \frac{e^{-\frac{1}{2}h^2(1+x^2)}}{(1+x^2)}dx$$

$$(h,g) \in \mathbb{R} \times \mathbb{R}. \quad (3)$$

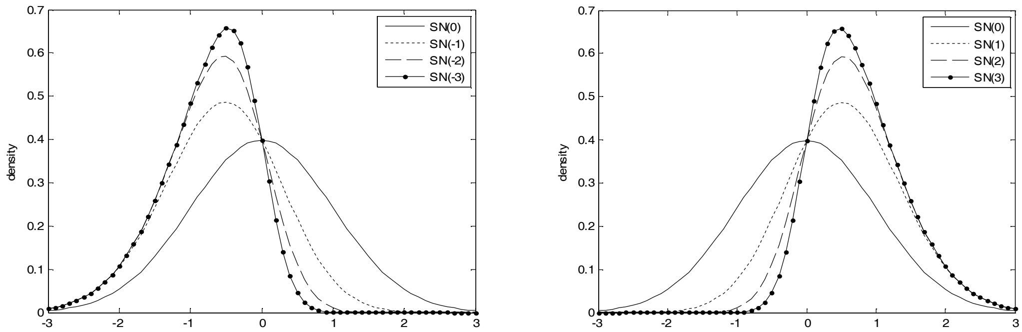


**Figure1.** Density functions of SN distribution for $\delta$s different.

***Remark2.1*** $1 - F_Z(-Z,\delta) = F_Z(Z,-\delta)$. Adding location and scale parameters to standard SN distribution, resulting in more flexibility and more control over the distribution of the value of this parameter can be found. With transformation$X = \mu + \sigma Z, \mu \in \Re, \sigma > 0$, PDF of $X$ is of the form (4), denoted by $X \sim SN(\mu, \sigma^2, \delta)$,

$$f_X(x) = \frac{2}{\sigma}\phi(\frac{x-\mu}{\sigma})\Phi(\delta\left(\frac{x-\mu}{\sigma}\right)). \quad (4)$$

The CDF, mean and variance of random variable $X$, respectively, is given by

$$F_X(X;\mu,\sigma^2,\delta) = \Psi(X;\mu,\sigma^2,\delta) = \Phi\left(\frac{X-\mu}{\sigma}\right) - 2T\left(\frac{X-\mu}{\sigma},\delta\right),$$
$$E(X) = \mu^* = \mu + \sigma\mu_z, \quad (5)$$
$$V(X) = \sigma^* = \sigma^2(1-\mu_z^2),$$

Where $\gamma = \frac{\delta}{\sqrt{1+\delta^2}}$, $\mu_z = \sqrt{\frac{2}{\pi}}\,\gamma$.

***Remark2.2*** A random variable X is written based on the location and scale parameters. But can be re-parameters, the random variable X to be rewritten as the following:

$$X = \mu^* + \sigma^* Z^*, \qquad Z^* = \frac{Z-\mu_z}{\sqrt{1-\mu_z^2}}. \quad (6)$$

Where $Z^*$ is a standardized variable, $\mu^*$ and $\sigma^*$ denoted mean and the standars deviasion of $X$, respectivly. In other words, re-parameterization of the random variable X can be based on mean and standard deviation wrote. Proof of all properties mentioned in Azzalini [2].

## 2.2. Input-Orianted BCC Model

Suppose that $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})'$ and $y_j = (y_{1j}, y_{2j}, \dots, y_{rj})'$are the input and output vectors of $DMU_j$, $j = 1, 2, \dots, n$. One of the basic DEA models are used to assess the DMUs, BCC model is in the input-oriented by Banker et al. [3] is presented as follows:

$$\min \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{S} s_r^+)$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, \quad i = 1,2,\dots,m,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1,2,\dots,s, \quad (7)$$
$$\sum_{j=1}^{n} \lambda_j = 1,$$

$s_i^- \geq 0, \quad i = 1,2,\dots,m,$

$s_r^+ \geq 0, \quad r = 1,2,\dots,s,$

$\lambda_j \geq 0, \quad j = 1,2,\dots,n.$

***Definition2.2*** *(Efficiency)* $DMU_o$is efficient if and only if in optimal solution of model (7) we have:

1. $\theta^* = 1.$
2. All slacks equal zero.

## 3 Methodology

In this section with introduced stochastic BCC model and asymmetric error structure, we formulate a stochastic version of the model introduced in Sub-section 2.2.

### 3.1 Stochasic Input-Orianted BCC Model

Following Cooper et al. [6], assume that $X_j = \left(X_{1j}, X_{2j}, \dots, X_{mj}\right)'$ and $Y_j = \left(Y_{1j}, Y_{2j}, \dots, Y_{rj}\right)'$be random input and output related to $DMU_j$, $j = 1, 2, \dots, n.$ The probabilistic costrant form of BCC model with stochastic data is as follows:

$\theta^*(\alpha) = \min \theta$

$s.t \quad P(\sum_{j=1}^{n} \lambda_j X_{ij} \leq \theta X_{io}) \geq 1 - \alpha, \quad i = 1,2,\dots,m,$ (8)

$$P(\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{ro}) \geq 1 - \alpha, \quad r = 1,2,\dots,s$$

$\sum_{j=1}^{n} \lambda_j = 1,$

$\lambda_j \geq 0, \quad j = 1,2,\dots,n.$

Where in model (8), $P$ means "probability" and $\alpha$ is a level of error between 0 and1.

***Definition3.1*** *(Stochastic efficiency)*$DMU_o$ is stochastically efficient if and only if in optimal solution:

1. $\theta^* = 1.$
2. All slacks equal zero.

Following Khodabakhshi and Asgharian [13], by introduce the external slack variables; we can convert the inequality to equality constraints in model (8). Therefore, the corresponding stochastic version of the model (7) is given by

$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{S} s_r^+\right)$

$s.t. \quad P(\sum_{j=1}^{n} \lambda_j X_{ij} - \theta X_{io} \leq -s_i^-) = 1 - \alpha, \quad i = 1,2,\dots,m,$

$P(\sum_{j=1}^{n} \lambda_j Y_{rj} - Y_{ro} \geq s_r^+) = 1 - \alpha, \quad r = 1,2,\dots,s,$ (9)

$\sum_{j=1}^{n} \lambda_j = 1,$ Using the aforementioned properties of SN distribution, one can show that

$\bar{\xi}_{ij} \sim SN(-\frac{\mu_z}{\sqrt{1-\mu_z^2}}, \frac{1}{1-\mu_z^2}, \bar{\delta}_i),$ (12)

$\bar{\zeta}_{rj} \sim SN(-\frac{\eta_z}{\sqrt{1-\eta_z^2}}, \frac{1}{1-\eta_z^2}, \bar{\varepsilon}_r),$ (13)

Where $\mu_z = \sqrt{\frac{2}{\pi}} \frac{\bar{\delta_i}}{\sqrt{1+\bar{\delta_i}^2}}$ and $\eta_z = \sqrt{\frac{2}{\pi}} \frac{\bar{\varepsilon_r}}{\sqrt{1+\bar{\varepsilon_r}^2}}$

The following relations are resulted from expressions (9) - (13):

$X_{ij} \sim SN\left(\mu_{ij}, \sigma_{ij}^2, \bar{\delta}_i\right),$

$Y_{rj} \sim SN\left(\eta_{rj}, \tau_{rj}^2, \bar{\varepsilon}_r\right).$

Assume that $i$th input and $r$th output of every DMUs are uncorrelated, i.e. for every j ≠ k,

$$cov(X_{ij}, X_{ik}) = 0, \quad i = 1,2,\dots,m,$$
$$cov(Y_{rj}, Y_{rk}) = 0, \quad r = 1,2,\dots,s. \qquad (14)$$

Now according to equations (11) and (14), it can be considered a same error for all DMUs, i.e. $\bar{\xi}_i = \bar{\xi}_{ij}$ and $\bar{\zeta}_r = \bar{\zeta}_{rj}$, for $j = 1,2,\dots,n$, $i = 1,2,\dots,m$, and $r = 1,2,\dots,s$.

***Theorem3.1*** Deterministic equivalent of stochastic BCC model (9) with asymmetric error structure (11) and uncorrelation for $i$th input and $r$th output of every DMUs is as follows:

$$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{S} s_r^+\right)$$
$$s.t \ \sum_{j=1}^{n} \lambda_j \mu_{ij}^* + s_i^- - \Psi_i^{-1}(\alpha)\left(\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*\right) = \theta\mu_{io}^*, \ i = 1,2,\dots,m,$$
$$\sum_{j=1}^{n} \lambda_j \eta_{rj}^* - s_r^+ + \Psi_r^{-1}(\alpha)\left(\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \tau_{ro}^*\right) = \eta_{ro}^*, \quad r = 1,2,\dots,s,$$
$$\sum_{j=1}^{n} \lambda_j = 1, \qquad (15)$$
$$s_i^- \geq 0, \quad i = 1,2,\dots,m,$$
$$s_r^+ \geq 0, \quad r = 1,2,\dots,s,$$
$$\lambda_j \geq 0, \quad j = 1,2,\dots,n.$$

Where $\Psi$ is the CDF of a SN distribution and $\Psi^{-1}(\alpha)$, is its inverse. It should be noted that $\Psi_i^{-1}(\alpha)$ and $\Psi_r^{-1}(\alpha\,)$ achieved by PDF (12) with skewnees parameter $-\bar{\delta}_i$ and PDF (13), respectively.

***Proof:*** Let $H_i = \sum_{j=1}^{n} \lambda_j X_{ij} - \theta X_{io}$. Using properties of SN distribution, asymmetric error structure (11) and uncorrelation for $i$th input of every DMUs we have:

$$H_i = \left(\sum_{j=1}^{n} \lambda_j \mu_{ij}^* - \theta\mu_{io}^*\right) + \bar{\xi}_i\left(\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*\right). \qquad (16)$$

Then

$$H_i \sim SN(a, b, \bar{\delta}_i), \qquad (17)$$

where

$$a = \sum_{j=1}^{n} \lambda_j \mu_{ij}^* - \theta\mu_{io}^* - \frac{\mu_z}{\sqrt{1-\mu_z^2}}\left(\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*\right),$$
$$b = \frac{1}{1-\mu_z^2}\left(\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*\right)^2.$$

From the first constraint in model (9), we will have

$$P\left(\frac{H_i - (\sum_{j=1}^{n} \lambda_j \mu_{ij}^* - \theta\mu_{io}^*)}{\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*} \leq \frac{-s_i^- - (\sum_{j=1}^{n} \lambda_j \mu_{ij}^* - \theta\mu_{io}^*)}{\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*}\right) = 1 - \alpha, \qquad (18)$$

however

$$\frac{H_i - (\sum_{j=1}^{n} \lambda_j \mu_{ij}^* - \theta\mu_{io}^*)}{\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*} \sim SN\left(-\frac{\mu_z}{\sqrt{1-\mu_z^2}}, \frac{1}{1-\mu_z^2}, \bar{\delta}_i\right). \qquad (19)$$

Using Remark 2.1 and necessary calculations, the stochastic constraint (18) can be converted to the following deterministic equivalent:

$$\sum_{j=1}^{n} \lambda_j \mu_{ij}^* + s_i^- - \Psi_i^{-1}(\alpha)\left(\sum_{j=1}^{n} \lambda_j \sigma_{ij}^* - \theta\sigma_{io}^*\right) = \theta\mu_{io}^*, \qquad (20)$$

Where $\Psi_i^{-1}(\alpha)$ calculated by PDF (12) with skewnees parameter $-\bar{\delta}_i$.

Similarly, let $K_r = \sum_{j=1}^{n} \lambda_j Y_{rj} - Y_{io}$, then using properties of SN distribution, asymmetric error structure (11) and uncorrelation for and $r$th output of every DMUs we will have:

$$K_r = \left(\sum_{j=1}^{n} \lambda_j \eta_{rj}^* - \eta_{ro}^*\right) + \tilde{\zeta}_r \left(\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \theta \tau_{ro}^*\right) \sim SN(c, d, \bar{\varepsilon}_r), \quad (21)$$

where

$$c = \sum_{j=1}^{n} \lambda_j \eta_{rj}^* - \eta_{ro}^* - \frac{\eta_z}{\sqrt{1-\eta_z^2}} \left(\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \tau_{ro}^*\right),$$

$$d = \frac{1}{1-\eta_z^2} \left(\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \tau_{ro}^*\right)^2.$$

As a result of the second constraint of the model (9) we have:

$$P\left(\frac{K_r - (\sum_{j=1}^{n} \lambda_j \eta_{rj}^* - \eta_{ro}^*)}{\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \tau_{ro}^*} \leq \frac{-s_i^- - (\sum_{j=1}^{n} \lambda_j \eta_{rj}^* - \eta_{ro}^*)}{\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \tau_{ro}^*}\right) = 1 - \alpha, \quad (22)$$

however

$$\frac{K_r - (\sum_{j=1}^{n} \lambda_j \eta_{rj}^* - \eta_{ro}^*)}{\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \tau_{ro}^*} \sim SN\left(-\frac{\eta_z}{\sqrt{1-\eta_z^2}}, \frac{1}{1-\eta_z^2}, \bar{\varepsilon}_r\right), \quad (23)$$

Therefore, $r$th output constraint of model (9) can be converted to

$$\sum_{ja=1}^{n} \lambda_j \eta_{rj}^* - s_r^+ + \Psi_r^{-1}(\alpha)\left(\sum_{j=1}^{n} \lambda_j \tau_{rj}^* - \tau_{ro}^*\right) = \eta_{ro}^*, \quad (24)$$

Where $\Psi_r^{-1}(\alpha)$ calculated by PDF (13). Thus, by (20) and (24), the deterministic model is completely specified. ■

**Remark3.1** In the model (15), if we assume $\bar{\delta}_i = \bar{\varepsilon}_r = 0, i = 1,2,\dots,m., r = 1,2,\dots,s$ , Then $\Psi_i^{-1}(\alpha) = \Psi_r^{-1}(\alpha) = \Phi^{-1}(\alpha)$ $i = 1,2,\dots,m., r = 1,2,\dots,s$.

***Theorem3.2*** Model (15) for every α level is feasible.

***Proof:*** Let $\lambda_o = 1, \theta = 1, \lambda_j = 0, j \neq o$ and also $s_i^- = s_r^+ = 0, i = 1,2,\dots,m,\ r = 1,2,\dots,s$. ■

## 4 Numerical Example

In this section, we use our proposed model to measuring of DMUs with a numerical example. Suppose that 10 DMUs include two inputs and one outputs and with stochastic inputs and outputs. These data have been indicated in Table1.

Therefore in asymmetric error structure, we suppose $\overline{\delta_1} = 0, \overline{\delta_2} \approx 1$ and $\overline{\varepsilon_2} \approx 2$. By running model (15) stochastic efficient all firms for $\alpha = 0.1, 0.05, 0.01$ gathered in Table2. Table 2 shows that if $\mathrm{DMU_o}$ is efficient in $\alpha'$ level of error, it can be efficient for each $\alpha < \alpha'$ and also, if $\mathrm{DMU_o}$ is inefficient in $\alpha'$ level of error, then it can be inefficient for each $\alpha' < \alpha$ .

**Table1:** Inputs and outputs

| $X_{1j}$ | $N(\mu,\sigma^2)$ | $X_{2j}$ | $SN(\mu,\sigma^2,\delta)$ | $Y_{1j}$ | $SN(\eta,\tau^2,\varepsilon)$ |
|---|---|---|---|---|---|
| $X1,1$ | $N(18789,885481)$ | $X2,1$ | $N(9.11,0.0004,1)$ | $Y1,1$ | $SN(49.6,48.02,1.9823)$ |
| $X1,2$ | $N(44304,6400900$ | $X2,2$ | $N(10.59,0.5329,1.02)$ | $Y1,2$ | $SN(73.13,13.1,2.1129)$ |
| $X1,3$ | $N(19729,2765569$ | $X2,3$ | $N(6.71,0.8649,0,98)$ | $Y1,3$ | $SN(108.04,225.6,1.9656)$ |
| $X1,4$ | $N(17435,1223236$ | $X2,4$ | $N(11.91,0.3025,0.94)$ | $Y1,4$ | $SN(44.97,13.76,2.0087)$ |
| $X1,5$ | $N(10379,210681)$ | $X2,5$ | $N(7.02,0.0225,1.043)$ | $Y1,5$ | $SN(31.63,38.94,2.0201)$ |
| $X1,6$ | $N(1667,1085764)$ | $X2,6$ | $N(18.99,0.8836,0.88)$ | $Y1,6$ | $SN(71.98,70.06,2)$ |
| $X1,7$ | $N(25462,1869689$ | $X2,7$ | $N(11.16,0.0081,0.9)$ | $Y1,7$ | $SN(78.05,195.72,1.9278)$ |
| $X1,8$ | $N(123064,426409$ | $X2,8$ | $N(15.05,0.4761,1.07)$ | $Y1,8$ | $SN(219.69,375.58,1.9292)$ |
| $X1,9$ | $N(36160,3837681$ | $X2,9$ | $N(8.87,0.3844,1.099)$ | $Y1,9$ | $SN(86.25,48.3,1.9934)$ |
| $X1,10$ | $N(46412,5317636$ | $X2,1($ | $N(19.88,0.25.0.89)$ | $Y1,10$ | $SN(194.58,1776.62,2.0194$ |

**Table2:** Stochastic efficiency of firms

| DMUs | $\alpha = 0.1$ | $\alpha = 0.05$ | $\alpha = 0.01$ |
|---|---|---|---|
| DMU1 | 0.7547 | 0.7645 | 0.7796 |
| DMU2 | 1 | 1 | 1 |
| DMU3 | 1 | 1 | 1 |
| DMU4 | 0.6498 | 0.6528 | 0.6787 |
| DMU5 | 0.8281 | 0.8458 | 0.856 |
| DMU6 | 1 | 1 | 1 |
| DMU7 | 1 | 1 | 1 |
| DMU8 | 1 | 1 | 1 |
| DMU9 | 0.9882 | 0.9882 | 1 |
| DMU10 | 1 | 1 | 1 |

## 5. Conclusion

DEA is one non-parametric technique for efficiency measure set of DMUs. In real world application, managers may encounter the data which are not deterministic but may be they are stochastic. The science of probability can assign a probability distribution for data but unsuitable choice of distribution leads to wrong inference. In this paper by introduce asymmetric error structure, we provided a stochastic BCC model for measuring the stochastic efficiency of DMUs with inputs and outputs having SN distribution. Also, in Remark 3.1 we showed that the model encompasses BCC model assuming a normal distribution of data as well. In order to future researches, applying asymmetric error structure in other DEA models is an interesting.

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