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A New Continuous Multi-State Reliability Model with Time Dependent Component Performance Rate

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Abstract
A CSS[1] is a system with the continues-state components. When a component has the ability to obtain all the situations from completely working to completely failed, it named continues-state component. In the real world, performance rate of elements are continuous and decrease by time. Continuity of components causes infinite working states and grows up the system states. In this paper we propose a new method for series-parallel continues-state RAP[2] using UGF[3] for multi-state systems. In this method at first we consider a binary CFR[4] system. Using Weibull distribution function for the performance rate of working state, this system upgraded to a CSC. Then the UGF for a series-parallel system has been studied and a numerical example presented to illustrate the reliability and availability computation.

Keywords: Continues-state system; Universal generating function; Reliability; Availability; performance level; Series-parallel; Structure function.

*. Corresponding author: Email: shahriari@iau.ae
1. Continues-State System
2. Redundancy Allocation Problem
3. Universal Generation Function
4. Constant failure Rate

1- introduction

Considering the communications between various fields of science and the science development, increasing the performance and efficiency and decreasing the costs of the systems are noticed more than ever. To achieves to a more reliable and efficient system, the reliability problems have more application. For drawing near these problems to the real world, different methods and techniques have been used. The improvement of these methods causes the more realistic and more precise solutions.

Now a day, many operational techniques have been designed for doing the different tasks in different environment. One of these techniques is categorizing based on performance states. Reliability systems divided in two main categories: binary states and multi-states. In traditional reliability theory, it assumed that the system and its components only have two states: working and failed. In fact, most of the applied systems have more than two states. They can have the numerous states between working state and failed one. These systems are known as MSS[1] [22].

Basic concepts of MSS can be found in [1, 17 and 18]. A brief study of reliability theory of MSS and its applications is in [15]. Ushakov [20] presented the UGF method for increasing the computation complexity of large scale MSS. Theory expansion and applications of UGF are presented in [8, 15, 19 and 21] and more detail description is on [11] which present the latest results on UGF. Output operational distribution function of a MSS can be determined using UGF. Also by combination of different operators, performance distribution of MSS with series, parallel, series-parallel and bridge structure have been studied on [12, 13 and 16]. UGF is a rapid method for determining the reliability of a MSS system, so it is an appropriate technique for multi-state optimization models. Levitin and Lisnianski [13], presented a optimization algorithm for MSS using UGF with series-parallel systems. Levitin [10], works on optimal location of multi-state components on a graph. Kuo and Wan [9], summarized the optimal location problems for increasing the reliability. UGF is the basic technique for evaluating the reliability of MSS in this study.

Each component in MSS basically has two futures: performance rate and its probability and both of them are deterministic. Calculation and prediction of these performance rate and probability

1. Multi State Systems

is very difficult [7]. The reasons of these difficulties are the simplification assumptions and invalidity of the data. By using simplification assumptions, the performance states of components decreases and it prevents the system states from increasing. This approach is practicable on many systems in real world [15]. The main incompetency of this approach is that some performance rates have been eliminated and this model can't represent the real operational behaviors of the system.

In recent studies, some methods were presented to make the problems closer to the real world. Fuzzy set theory is one of the useful tools for evaluating the MSS reliability with uncertain probabilities or uncertain performance rates [23]. Ding and Lisnianski [24], proposed a fuzzy UGF for evaluating the MSS reliability with fuzzy probabilities and fuzzy performance rates.

In most studies, the performance rates of components are considered as described rates. But in most real systems, the performance rates of components are continuous. In [2, 3 and 4], there is a new type of CSF[1] that permits the components to have all positions between completely working state and failed state presented. Block and Savits [2] and Baxter and Kim [5], decomposed the CSF to BSF[2] and results were expressed as the system performance range. Stochastic behavior of CSS and its components have been discussed by Brunelle and Kapur [6], in this situation, elements have a combined discrete and continuous performance function. Lisnianski [14] presented the method of forecasting boundary points of reliabilities in a discrete-continuous system using UGF method.

In this paper a new method for reliability and availability evaluation of series-parallel CSS was presented. The performance rates of components in this model are continuous and they have their own probabilities. The performance rates and corresponding probabilities decrease by time. In section 2 the system definitions have been introduced. A numerical example has been presented in section 3 to illustrate the method calculations and the section 4 deals with conclusion and further studies.

1. Continuous structure functions
2. Binary structure functions

Nomenclature

j :	Elements index for CSS, $j = 1, 2, \ldots, n$
k_j :	Number of different discrete states for j^{st} elements that $k_j = 2$
i :	Index of discrete CSS, $i = 1, 2$
l :	Sub-system index, $l = 1, 2, \ldots, s$
n_l :	Elements number of sub-system number l
K :	All available combinations of elements discrete states
$\{G_{j1}(t), G_{j2}(t)\} = G_j(t)$:	Set of time dependent performance rate for element number j
$\{P_{j1}(t), P_{j2}(t)\} = P_j(t)$:	Set of time dependent probability for element number j
$X(t)$:	Continuous performance rate for all CSS elements
β :	Weibull c.d.f shape parameter for performance rate $X(t)$
η :	Weibull c.d.f scale parameter for performance rate $X(t)$
$P_{j1}(t) = R_j(t)$:	Time dependent probability function for working element in discrete system
$P_{j2}(t) = F_j(t)$:	Time dependent probability function for failed element in discrete system
λ_j :	Exponential c.d.f parameter of j th element
ϕ :	System structure function
Ω_ϕ :	UGF combination operator
ϕ_s :	Series elements system structure function
ϕ_p :	Parallel elements system structure function
ω :	System demand
$A(\omega, t :)$	System availability level at time t for system demand ω
σ_A :	Availability level operator
$U(Z, t)$:	UGF for CSS

Other parameters will be introduced when needed.

2- System definition

The discussed problem is a continuous reliability model with series-parallel structure. It means that the system has some series of subsystems and each sub-system has some parallel components that have to be determined. Performance state of each component is continuous and the corresponding probability of each state is continuous and time-dependent. At the start of system's operation, all the components are working and as time passes, the probability and performance rate of each state decreases. The components are non repairable and the strategy of sub-systems are active. This system is a generalized model of a MSS using UGF [15]. If $u_j(Z)$ considered as UGF of j^{st} component, then we have:

$$u_j(z) = \sum_{i=1}^{k_j} p_{ij}.z^{g_{ji}} \tag{1}$$

For calculation of output distribution function for MSS with ϕ system structure function, the Ω_ϕ operator can be used as follow:

$$U(Z) = \Omega_\phi\{u_1(z), u_2(z), \ldots, u_n(z)\} = \Omega_\phi\left\{\sum_{i_1=1}^{k_1} p_{1i_1}.Z^{g_{1i_1}}, \sum_{i_2=1}^{k_2} p_{2i_1}.Z^{g_{2i_1}}, \ldots, \sum_{i_n=1}^{k_n} p_{ni_1}.Z^{g_{ni_1}}\right\} = \sum_{i_1=1}^{k_1}\sum_{i_2=1}^{k_2}\cdots\sum_{i_n=1}^{k_n}\left\{\prod_{j=1}^{n} p_{ji_j}.Z^{\phi(g_{1i_1}, g_{1i_2}, \ldots g_{1i_n})}\right\} \tag{2}$$

In equation (2), g_{ji_j} is performance state of j^{th} component and P_{ji_j} is the corresponding probability of this state and both of them are constant.

In this paper, the number of performance states of each component is equal 2, $(i = \{1,2\})$. It means that each component is working or failed and performance rate of working state decreases by passing of the time. So the continuous performance probability has two situations that illustrate by P_{j1} for working state P_{j2} and for failed state. j^{st} working component has λ_j constant failure rate. It means that component life distribution function is exponential. For every λ_j we know that $\{R_j(t) + F_j(t) = 1\}$, so we have:

$$P_{j1} = R_j(t) = e^{\lambda_j t} \tag{3}$$

$$P_{j2} = F_j(t) = 1 - e^{\lambda_j t} \tag{4}$$

By combining the equations (2), (3) and (4), the u_j can be calculated as follow:

$$u_j(z,t) = \sum_{j=1}^{k_j} P_{ji_j}(t).z^{G_{ji_j}(t)} = R_j(t)z^{G_{ji_j}(t)} + F_j(t)z^0 \tag{5}$$

The performance rate of each component decreases by working, so we consider the components performance states as:

$$G_{ji_j}(t) = g_{ji_j}.X_j(t) \tag{6}$$

In equation (6), $G_{ji_j}(t)$ is divided into two sections: g_{ji_j} is a discrete performance rate and $X_j(t)$ is a continuous function. For j^{st} component, g_{ji_1} is deterministic and can be different from a component to another one, and $g_{ji_2}=0$.

The performance rate function $X_j(t)$, has to have some characteristics to represent its behavior in the real world. Generally, the performance rate of equipments and components decreases by passing of the time. For example, a water pump works perfectly, when it is new and after a while, the amount of pumped water decreases. In this situation, we prefer to find a distribution to provide three conditions:

1. At the start of working time, performance rate is in highest level,
2. Performance rate decreases slowly after a long working time,
3. After this time, performance rate decreases rapidly and the component tends to move towards the failed state.

The normal and Weibull distributions functions have these conditions and we use Weibull distribution function

I Weibull distribution, when $\gamma=0$ and $3\le\beta\le4$, the p.d.f is very similar to normal p.d.f, so we can use this function instead of normal.

Now we must define the amount of parameter η to achive to presente second condition. If $\hat{t}$ is as considered the time of the performance rate breakdown, then η can be calculated as follows:

$$\eta=-\frac{\hat{t}}{\{Ln\,X(\hat{t})\}^{1/\beta}} \tag{7}$$

So, before $\hat{t}$, performance rate decreases slowly and after $\hat{t}$ performance rate decreaes rapidly and tends to zero. Performance functions of identical working elements are:

$$X_j(t)=X(t)=e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{8}$$

And:

$$U(Z,t)=\Omega_\phi\{u_1(z,t),\ldots,u_n(z,t)\}=$$
$$\Omega_\phi\left\{\sum_{i_1=1}^{k_1=2}p_{1i_i}(t).Z^{G_{1i_1}(t)},\ldots,\sum_{i_n=1}^{k_n=2}p_{ni_n}(t).Z^{G_{ni_n}(t)}\right\}= \tag{9}$$
$$\sum_{i_1=1}^{k_1=2}\sum_{i_2=1}^{k_2=2}\cdots\sum_{i_n=1}^{k_n=2}\left\{\prod_{j=1}^{n}P_{ji_j}(t).Z^{\phi\{G_{1i_1}(t),\ldots,G_{ni_n}(t)\}}\right\}$$

To find the system structure function $\phi\{G_{1i_1}(t),\ldots,G_{ni_n}(t)\}$ for a series-paralle system, we trace these two steps:

Step 1: for each parallel sub-system, calculate system structor function ϕ_P. Each system has $l=\{1,2,\ldots,s\}$ sub-systems and the number of sub-systems is eqaul to n_l and $\sum_{l=1}^{s}n_l=n$. So, for each parallel sub-system we have:

$$\begin{matrix} \phi_P\{G_1,\ldots,G_{n_1}\} \quad ; \quad l=1 \\ \phi_P\{G_{n_1+1},\ldots,G_{n_1+n_2}\} \quad ; \quad l=2 \\ \vdots \\ \phi_P\{G_{n_1+\ldots+n_{s-1}},\ldots,G_{n_1+\ldots+n_s}\} \quad ; \quad l=s \end{matrix} \tag{10}$$

In equation (10), G_{n_l} is the continuous performance level of $j=n_l$ component from l^{st} sub-system.

Step 2: calculate the system structure function for all parallel sub-systems that are serialy linked to gether as:

$$\phi_s=\begin{Bmatrix} \phi_P\left(G_1,\ldots,G_{n_1}\right), \\ \phi_P\left(G_{n_1+1},\ldots,G_{n_1+n_2}\right),\ldots, \\ \phi_P\left(G_{n_1+\ldots+n_{s-1}},\ldots,G_{n_1+\ldots n_s}\right) \end{Bmatrix} \tag{11}$$

The system structure function of a parallel system ϕ_P, is presented in [15] as sum of the sub-system components as follow:

Also, the system structure function of a series system ϕ_P, is presented in [15] as minimum of the sub-system components as follows:

$$\phi_s(G_1,\ldots,G_n)=Min(G_1,\ldots,G_n) \tag{12}$$

Acoording to equations (15) and (16), system total ϕ is calculated as:

$$\phi_T=Min\begin{Bmatrix} \left(G_1+\ldots+G_{n_1}\right), \\ \left(G_{n_1+1}+\ldots+G_{n_1+n_2}\right),\ldots, \\ \left(G_{n_1+\ldots+n_{s-1}}+\ldots+G_{n_1+\ldots n_s}\right) \end{Bmatrix} \tag{13}$$

By combining equations number (6), (7), (8) and (13), the $U(Z,t)$ is:

$$=\sum_{i_1=1}^{k_1=2}\sum_{i_2=1}^{k_2=2}\cdots\sum_{i_n=1}^{k_n=2}\left\{\prod_{j=1}^{n}P_{ji_j}(t).Z^{e^{-\left(\frac{t}{\eta}\right)^{\beta}}.Min\begin{Bmatrix} \left(g_1+\ldots+g_{n_1}\right), \\ \left(g_{n_1+1}+\ldots+g_{n_1+n_2}\right),\ldots, \\ \left(g_{n_1+\ldots+n_{s-1}}+\ldots+g_{n_1+\ldots+n_s}\right) \end{Bmatrix}}\right\} \tag{14}$$

State space for a CSS can be divided to acceptable and unacceptable sets. It depends on the relations between CSS performance and the damand level that is determined by decision makers.

The relation between system parformance and demand level ω is presented in [23] as system state eligibility index as:

$$r_j=g_j-\omega \tag{15}$$

The i^{st} state is acceptable if and only if $r_j\geq 0$.

vilability for a CSS is the probability that system remains at acceptable sub-set. In CSS, availability is defined as probability of CDD performance rate is equal or grater then demand level ω as follow:

$$A(\omega)=\sum_{r_j\geq 0}p_j \quad , \quad r_j=g_j-\omega \tag{16}$$

We can calculate $U(Z,t)$ fusing or a CSS δ_A operator as:

$$A(\omega)=\delta_A\{U(z,t),\omega\}=$$

$$\delta_A\left\{\sum_{i_1=1}^{k_1=2}\sum_{i_2=1}^{k_2=2}\cdots\sum_{i_n=1}^{k_n=2}\prod_{j=1}^{n}P_{ji_j}(t).Z^{e^{-\left(\frac{t}{\eta}\right)^{\beta}}.Min\begin{Bmatrix} \left(g_1,\ldots,g_{n_1}\right), \\ \left(g_{n_1+1},\ldots,g_{n_1+n_2}\right),\ldots, \\ \begin{pmatrix} g_{n_1+\ldots+n_{s-1}},\ldots, \\ g_{n_1+\ldots+n_s} \end{pmatrix} \end{Bmatrix}}\right\}= \tag{17}$$

$$\sum_{i=1}^{k}P_i.\alpha_i \quad ; \quad \alpha_i=\begin{cases} 0 & ; \quad r_i\geq 0 \\ 1 & ; \quad r_i<0 \end{cases}$$

In equation (21), $K=\prod_{j=1}^{n}k_j$ are the total descrete system states [15]. In the next section a numerical example will be presented to demonstrate the stage of model in obtaining the system availability.

3- numerical example

assume a pumping system that contains 2 different pump stations and each pump station has two differet pumps. System structure is presented in figure 1.

The Water flow is the performance rate level that it is continuous and time-dependent. Each pump in each station has two states: completly failed and working (depending on time). Performance rate for these four components are $g_{11}=4, g_{21}=3, g_{31}=2, g_{41}=5$. Corresponding probabilities has exponential distributions. The failure rate of the pumps are $\lambda_1=0.0001, \lambda_2=0.0003, \lambda_3=0.0002, \lambda_4=0.0004$. We assume that pump works for 200^h and in this time performance rate of each pump decreaces by 1%, so we have $\left(X(\hat{t}=200)=0.99\right)$ and after this time performance rate of each pum decreases rapidly. Shape parameter is considered as $\beta=3$ and scale parameter can be calculated using Using equation (7)For this example the opproximate value of η is near $\eta=1000$. The UGF for each component is as follows:

$$\begin{aligned} u_1(z,t) &= e^{-t/0.0001}.Z^{-4\left(t/1000\right)^3}+\left(1-e^{-t/0.0001}\right).Z^0 \\ u_2(z,t) &= e^{-t/0.0003}.Z^{-3\left(t/1000\right)^3}+\left(1-e^{-t/0.0003}\right).Z^0 \\ u_3(z,t) &= e^{-t/0.0002}.Z^{-2\left(t/1000\right)^3}+\left(1-e^{-t/0.0002}\right).Z^0 \\ u_4(z,t) &= e^{-t/0.0004}.Z^{-5\left(t/1000\right)^3}+\left(1-e^{-t/0.0004}\right).Z \end{aligned} \quad (18)$$

We considr $t=100$ for this problem. So $R_j(100)=\{0.9900\,,\,0.9704\,,\,0.9802\,,\,0.9607\}$ and $F_j(100)=1-R_j(100)$ and the $u_j(z,t)$ are:

$$\begin{aligned} u_1(z,100) &= 0.99.Z^{3.996}+0.01.Z^0 \\ u_2(z,100) &= 0.9704.Z^{2.997}+0.0296.Z^0 \\ u_3(z,100) &= 0.9802.Z^{1.998}+0.0198.Z^0 \\ u_4(z,100) &= 0.9607.Z^{4.995}+0.0393.Z^0 \end{aligned} \quad (19)$$

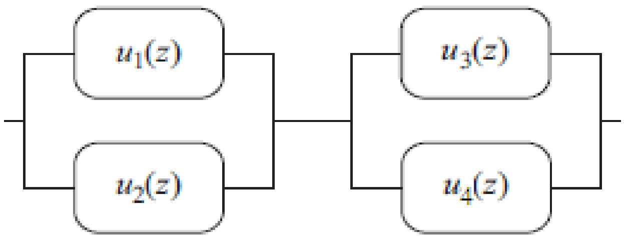

Fig 1. System state space

Using Ω_{ϕ} operator and series-paralle structor presented in figure 3, system structor function is

$$Min\{G_{1i_1}(t)+G_{2i_2}(t), G_{3i_3}(t)+G_{4i_4}(t)\}$$

$$= X(t).Min\{g_{1i_1}(t)+g_{2i_2}(t), g_{3i_3}(t)+g_{4i_4}(t)\}$$

The system output probability function is calculated as:

$$\Omega_{\phi}\{u_1(z,t), u_2(z,t), u_3(z,t), u_4(z,t)\} =$$

$$\Omega_{\phi_s}\{\Omega_{\phi_p}\{u_1(z,t), u_2(z,t), \Omega_{\phi_p} u_3(z,t), u_4(z,t)\}\} =$$

$$\Omega_{\phi_s}\left\{\begin{matrix}(0.9607.Z^{6.993}+0.0293.Z^{3.996}+0.0097.Z^{2.997}+0.0003.Z^{0}),\\(0.9417.Z^{6.993}+0.0385.Z^{1.998}+0.019.Z^{4.994}+0.0008.Z^{0})\end{matrix}\right\} = \quad (20)$$

$$0.9047.Z^{6.993}+0.0183.Z^{4.995}+0.0281.Z^{3.996}+$$

$$0.0093.Z^{2.997}+0.0385.Z^{1.998}+0.0011.Z^{0}$$

Because of the silmilar states, the total system states reduced to 6. If the demand level was assumed as $\omega = 3$, CSS would be calculated as follow:

$$A(3) = \delta_A\{U(z,100), 3\} =$$

$$\delta_A\begin{pmatrix}0.9047.Z^{6.993}+0.0183.Z^{4.995}+0.0281.Z^{3.996}+\\0.0093.Z^{2.997}+0.0385.Z^{1.998}+0.0011.Z^{0}\end{pmatrix} = \quad (21)$$

$$\sum_{i=1}^{6} p_1.\alpha_i = 0.9047+0.0183+0.0281+0+0+0 = 0.9511$$

4- conclution and furtur studies

In CSS reliability model survey, the components have extreme time-dependent states and it causes the extreme system states. These systems have a large state space and reliability and availability evaluation is very difficault for them. In this paper we presented a new method for calculating the reliability of time dependent CSS that reduced the computation time and easily reached an exact solution.

In presented method, at first the components state were divided to working and failed states. Then by generalizing the UGF method for multi-state system, the working state transformed to a continuous state. To have access to continuous and time-dependent perforamance states, we design the performance rate function. This functions were obtained using Weibull distribution function that is similar to normal distribution function. Each component was considered as a CFR component. Then availabilty of the system was calculated by finding the UGF function for output probability function for a continuous and time-dependent state system. We tried to make the raliabilty problems closer to the rael world. The results of this paper can be used in reliability optimization problems like RAP.

References

[1] R. Barlow, A. Wu, Coherent systems with multi-state elements. Mathematical and Operation Research, 3 (1978) 275–281.

[2] Block HW, Savits TH. Continuous multi-state structure functions. Operation Research, 1984; 32: 703–14.

[3] Baxter LA. Countinuum structures I. J Appl Prob 1984; 21:802–15.

[4] Baxter LA. Countinuum structures II. Math Proc Cambridge Philos Soc 1986; 99:331–8.

[5] Baxter LA, Kim C. Bounding the stochastic performance of continuum structure functions I. J Appl Prob 1986; 23:660–9.

[6] Brunelle R, Kapur KC. Countinuous-state system reliability: an interpolation approach. IEEE Trans Reliab 1998; 47:181–7.

[7] C.H. Cheng, D.L. Mon, Fuzzy system reliability analysis by interval of confidence, Fuzzy Sets and Systems 56 (1993) 29–35.

[8] B. Gnedenko, I. Ushakov, Probabilistic Reliability Engineering, Wiley, NY, Chichester, Brisbane, 1995.

[9] W. Kuo, R. Wan, Recent advances in optimal reliability allocation, IEEE Trans. Systems, Man Cybernet. Part A: Systems Humans 37 (2007) 143–156.

[10] G. Levitin, Optimal allocation of multi-state retransmitters in acyclic transmission network, Reliability Eng. System Safety 75 (2002) 73–82.

[11] G. Levitin, Universal Generating Function and its Applications, Springer, Berlin, 2005.

[12] G. Levitin, A. Lisnianski, Joint redundancy and maintenance optimization for multi-state series-parallel systems, Reliability Eng. System Safety 64 (1998) 33–42.

[13] G. Levitin, A. Lisnianski, H. Ben-Haim, D. Elmakis, Redundancy optimization for series-parallel multi-state systems, IEEE Trans. Reliability 47 (2) (1998) 165–172.

[14] A. Lisnianski, Estimation of boundary points for continuum-state system reliability measures, Reliability Engineering and System Safety 74 (2001) 81–88.

[15] A. Lisnianski, G. Levitin, Multi-state System Reliability Assessment, Optimization, Applications, World Scientific, Singapore, 2003.

[16] A. Lisnianski, G. Levitin, H. Ben-Haim, Structure optimization of multi-state system with time redundancy, Reliability Eng. System Safety 67 (2000) 103–112.

[17] J. Murchland, Fundamental concepts

and relations for reliability analysis of multi-state systems and fault tree analysis, Theoretical and Applied Aspects of System Reliability, SIAM, Philadelphia, 1975, pp. 581–618.

[18] E. El-Neveihi, F. Prochan, J. Setharaman, Multi-state coherent systems, J. Appl. Probab. 15 (1978) 675–688.

[19] K. Reinshke, I. Ushakov, Application of Graph Theory for Reliability Analysis, Radio i Sviaz, Moscow, 1988 (in Russian).

[20] I. Ushakov, Universal generating function, Soviet J. Comput. Systems Sci. 24 (5) (1986) 118–129.

[21] I. Ushakov, The method of generalized generating sequences, European J. Oper. Res. 125 (2) (2000) 316–323.

[22] Vesely W. principles of resource-effectiveness and regulatory-effectiveness for risk-informed applications: Reducing burdens by improving effectiveness. Rel. Engng & System saf1999; 63, 283-292.

[23] MA Nayebi, M Sharifi, MR Shahriari, O Zarabadipour, Fuzzy-chance constrained multi-objective programming applications for inventory control model, Applied Mathematical Sciences, Vol. 6, 2012, no. 5, 209 - 228

[24] Yi Ding, Anatoly Lisnianski. Fuzzy universal generating function for multi-state system reliability assessment, Sets and Systems 159 (2008) 307–324.