Available online at http://jnrm.srbiau.ac.ir
Vol.1, No.4, Winter 2016
ISSN:1682-0169

Journal of New Researches in Mathematics

Science and Research Branch (IAU)

A Method for Target Setting with Share Data

B. Rahmani Parchkolaei[a*], **Z. Moghaddas**[b]

[(a)] *Departments of Mathematics, Central Tehran Branch, Islamic Azad University, Tehran, Iran*
[(b)] *Departments of Mathematics, Qazvin Branch, Islamic Azad University,Qazvin, Iran*

Received Autumn 2015, Accepted Winter 2016

Abstract
Data Envelopment Analysis (DEA) is a mathematical programming technique for evaluating the relative efficiency of a set of Decision Making Units (DMUs) and can also be utilized for setting target. Target setting is one of the important subjects since according to its results efficiency can be increased. An important issue to be currently discussed, is to set target while considering share data. These data for each individual indicate the share of the unit, which takes part in an activity, from the whole amount which is a predefined constant. It is obvious that the sum of units' share is equal to the entire amount. Thus, any changes in the magnitude of these data has to be dependent on the changes in data of other units. In this paper a two-stage procedure is developed to find benchmark units where share data exist. The fact that all DMUs are jointly projected onto the new efficient frontier and simplicity, are the significant features of the proposed method. With a numerical example we demonstrate how this method works.

Keywords: *Data Envelopment Analysis, Target, Share Data*

*. Corresponding Author: Email: bijanrah40@gmail.com

1. Introduction

Data Envelopment Analysis as a mathematical programming technique is nowadays used for performance evaluation of a set of DMUs. There exist various DEA models on the basis of variant assumptions about production technology. One of the important key features of DEA is recognizing target units and reference set for those units do not perform efficiently. The most well known DEA models are referred as radial models which include CCR, first provided by Charnes et al. [4] and BCC, first introduced by Banker et al. [3].

In recent years, several studies have undertaken benchmarking and provided new methods in different circumstances. Some of the important papers are as follows. Post and Spronk [10] presented a procedure for benchmarking. In their approach they extended DEA to incorporate the interactive Multiple Goal Programming (IMGP) which is called Interactive Data Envelopment Analysis (IDEA). Gonzalez and Alvarez [6] tried to find an appropriate target for inefficient DMUs. They note that the most analogous efficient DMU is the better target to be imitated. Hougaard and Tvede [8] considered an axiomatic approach to benchmark selection. They considered simple and weak axioms and then defined a family of benchmark. Hougaard and Keiding [7] examined the benchmark selections existed in production factors and technology. They noted that without convexity in class of technologies, then continuous selections cannot exist. Cook et al. [5] presented mathematical programming models to be used in benchmarking. They used these models where multiple performance measures are required. Seo et.al [12] used the integrated form of DEA and decision tree to provide an approach for benchmarking. Presented approach allows the manager for selecting those processes help DMUs to be improved. Ross and Droge [11] provided a benchmarking process to be represented in a large supply chain system in which distribution centers exists. Aparicio et al. [1] proposed an approach in order to find the closest targets for a DMU to imitate. Their idea is upon this fact that closer targets cause minor variations in the data of the inefficient units. Trappey and Chiang [13] used DEA analysis for providing a benchmarking planning. In this work they tried to gain the maximal profit. Baek and Lee [2] studied the use of the least distance measure in order to obtain the shortest projection from the DMU to the strong efficient frontier. Thus, they let an inefficient DMU search for the easiest way to increase the efficiency score. Wu et al. [14] used cross efficiency evaluation method to measure the performance of DMUs. As mentioned above various method have been presented in literature for benchmarking and target setting. Considering share data" as one of the important types of data, used frequently in real life, here the aim is to present a new method based on this type of data. These data for each unit indicate the share of that unit from the total amount. Therefore the sum of all units' share is equal to the entire amount. Since the sum of this element among all units equals the predefined constant, any changes in the share of these units have to be dependent on each other. We turn into an example in stock market for indicating the issue more factual. Consider five stockholders each of which has a share in an specific stock. If one of them wants to increase its share, in this stock, the share of others must be decreased. Therefore, any changes in share of each of them entails changing in share of others. So, it is impossible to set target individually for each unit. In this paper considering this data type, a method is presented which identifies efficient targets. The current article proceeds as follows: In the next section, DEA background will be briefly reviewed. Then, in Section 3, the proposed method will be discussed. An illustrative example is documented in section 4 and section 5 concludes the paper.

2. DEA Background

Data enveloping analysis is now utilized frequently for performance evaluation of set of DMUs. Let DMU_o be a unit under assessment from a total n units. Define $x_o \in R^{m_+}$ and $y_o \in R^{s_+}$ as the inputs and outputs of DMU_o. Consider the most general definition of the production technology ,T_c, which is defined with a set of semipositive (x, y) as:

$$T_c = \{(x, y)| \quad x \geq \sum_{j=1}^{n} \lambda_j x_j, \quad y \leq \sum_{j=1}^{n} \lambda_j y_j, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n\} \tag{2.1}$$

The constant returns to scale form of the enveloping problem in input orientation which first introduced by Charnes et al.[4], is as follows:

$$\begin{aligned} & min \qquad \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s, \\ & \lambda_j \geq 0, \quad j = 1, \ldots, n. \end{aligned} \tag{2.2}$$

Considering DEA models it is possible to find an efficient frontier that can also be taken into account as a benchmark frontier. Thus, having the relative efficiency of eachDMU, it can be separately projected onto the efficient frontier. In order to find targets for DMU_o one should follow the two-stage optimization procedure. In the first stage the optimal value of the objective function of CCR model is obtained. This factor showed the proportional reduction in inputs without any alterations in outputs. Note that, with this factor shortfall in outputs or excess in input cannot be considered. In order to find the target the second stage should be solved for identifying the nonradial improvements for inputs and outputs. This aim is done in the following model by maximizing the sum of input and output slacks. The second stage is as follows:

$$\max \quad \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta^* x_{io}, \quad i = 1,\ldots,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1,\ldots,s,$$

$$\lambda_j \geq 0, \quad j = 1,\ldots,n, \qquad (2.3)$$

$$s^- \geq 0, \quad s^+ \geq 0$$

Therefore, targets for DMU_o will be $(\theta^* x_o - s^{-*}, y_o + s^{+*})$, in which θ^* is the input radial improvement and s^{-*} and s^{+*} are input excess and output shortfall, respectively.

3. Target Setting with Share Data

As mentioned in the previous section conventional DEA models set target for each DMU separately. Hence, in this section considering ßhare data" two models are presented for a two-stage procedure which, in a joint manner, find a target for each DMU. In general target setting, the data of different units do not have a special relation to one another, but share data indicate the share of a unit from the total amount which shows relation among the units. Since the sum of this element among all units equals the predefined constant, any changes in the amount have to be dependent on the changes in data of other units. Thus, while working with share data it is not possible to set target for each unit individually. Here, the aim is to carry out the analysis for benchmarking in such situations. The advantage of this model is that in lieu of solving an independent LP problem assessing each DMU in turn, all $DMUs$ are simultaneously assessed through solving an LP model.

Let, p refer to the number of $DMUs$ and y_{1j} for all j (j=1,...,n) indicates $sharedata$. In the presented model sum of the first element of output in all units is confined to be equal to the predefined constant "c". This predefined constant is

$\sum_{p=1}^{n} y_{1p}$. In this paper we restrict the analysis to the case in which initial sum of the specific element, in output vector, remains unchanged throughout the work. Constraint (a) in the presented models, (3.4) and (3.5), ensures that this scenario will happen. We consider this model in input orientation. The same analysis can be performed for output orientation with share data in output. Thus, for all p and j (p, j=1,...,n), the sum of $\lambda_j^p y_{1j}$ over p and j, is considered to be equal to the initial sum of the first element of output through all units. That means, any alterations in the first element of output in all units are under the influence of changes of this factor in other units in a way that the initial sum of this element remains unchanged.

In these models the corresponding slacks of the first element of all $DMUs$ are considered unrestricted in sign in order to enhance models to find suitable targets. The proposed procedure is the extension of two-stage CCR model in which share data are incorporated. The following model consists of n separable models that are considered in a joint manner. Since for all j the sum of y_{1j} after changes must equal the total amount; for all p, s_1^{+p} is considered unrestricted in sign.

$$
\begin{aligned}
\min \quad & \sum_{p=1}^{n} \theta_p \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j^p x_{ij} + s_i^{-p} = \theta_p x_{ip}, \\
& i = 1,\dots,m, p = 1,\dots,n \\
& \sum_{j=1}^{n} \lambda_j^p y_{rj} - s_r^{+p} = y_{rp}, \\
& r = 1,\dots,s, p = 1,\dots,n, \\
& \sum_{j=1}^{n} \lambda_j^1 y_{1j} + \dots + \sum_{j=1}^{n} \lambda_j^n y_{1j} = c, \quad (a) \\
& s_i^{-p} \geq 0, s_r^{+p} \geq 0, \\
& i = 1,\dots,m, r = 2,\dots,s, p = 1,\dots,n \\
& \lambda_j^p \geq 0, \quad j = 1,\dots,n, \\
& s_1^{+p} unrestricted, \quad p = 1,\dots,n.
\end{aligned}
\tag{3.4}
$$

This model is some how like the model discussed in Post and Spronk [11]. This paper and that of Post and Spronk consider n separable models in one LP model but the aim in these two models are different from each other since Post and Spronk [11] presented a procedure for performance benchmarking. They extended the DEA technique for incorporating the interactive multiple goal programming and called it interactive data envelopment analysis. Note that in this paper, constraint (a) and unrestricted s_1^{+p} are considered to set target for $DMUs$ in a joint manner while share data are taken into account. Now, for the second stage consider the following model in which the objective is to maximize the sum of slacks.

In the following model for all p, θ_p^* is the optimal solution of model (3.4). According to the constraint (a) in model (3.5), for all j any changes in y_{1j} are implemented in a way that the sum of the resultant output still equals the sum of initial amount.

$$\begin{aligned}
\max \quad & \sum_{p=1}^{n}\left(\sum_{i=1}^{m} s_i^{-p} + \sum_{r=2}^{s} s_r^{+p} + |s_1^{+p}|\right) \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j^p x_{ij} + s_i^{-p} = \theta_p^* x_{ip}, \\
& i = 1,\dots,m, p = 1,\dots,n, \\
& \sum_{j=1}^{n} \lambda_j^p y_{rj} - s_r^{+p} = y_{rp}, \\
& r = 1,\dots,s, p = 1,\dots,n, \\
& \sum_{j=1}^{n} \lambda_j^1 y_{1j} + \dots + \sum_{j=1}^{n} \lambda_j^n y_{1j} = c, \quad (a) \\
& \lambda_j^p \geq 0, \quad j = 1,\dots,n, \quad p = 1,\dots,n, \\
& s_i^{-p} \geq 0, \quad i = 1,\dots,m, \quad p = 1,\dots,n, \\
& s_r^{+p} \geq 0, \quad r = 2,\dots,s, \qquad (3.5) \\
& s_1^{+p} unrestricted, p = 1,\dots,n.
\end{aligned}$$

In the above model c defines as $\sum_{p=1}^{n} y_{1p}$. The obtained slacks through solving this model is greater than or equal to those resulted from model (3.4). For all p, each of s_1^{+p} is deemed unrestricted in sign thus in the optimal solution of model (3.5) by summing the constraints corresponding to the first element of output vector of all units, we always have $\sum_{p=1}^{n} s_1^{+*p} = 0$. We can draw the conclusion that if there exists a positive s_1^{+*p} then, there must exist another unit, like k, whose corresponding slack, s_1^{+*k}, is negative. This fact ensures that the changes through these units are related to each other. According to non radial changes which are resulted from model (3.5), the first element of output vector for all units is increased or decreased in such a way that finally after these changes the sum of this element in target units is equal to the sum of initial amounts. Therefore, a target for DMU_o is $(\theta^* x_o - s^{-*}, y_o + s^{+*})$, in which θ^* is the proportional changes and s^{-*} and s^{+*} are non radial changes, respectively. This model can be easily converted into its linear counterpart. To this end let $s_1^{+p} = u^p - v^p, u^p \geq 0, v^p \geq 0$ for all p, where

$$u^p = \begin{cases} 0, & s_1^{+p} \leq 0, \\ s_1^{+p}, & s_1^{+p} \geq 0. \end{cases} \quad v^p = \begin{cases} 0, & s_1^{+p} \geq 0, \\ -s_1^{+p}, & s_1^{+p} \leq 0, \end{cases}$$

Which results in $u^p . v^p = 0$ for all p. Now, accordingly $|s_1^{+p}| = u^p + v^p$ for all p. It should be noted that by using this variable transformation the nonlinear constraint, $u^p . v^p = 0$, is also added to the model. But the aforesaid nonlinear constraint is redundant due to the dependency of corresponding coefficient column vectors. Thus, model (3.5) can be easily written in linear form. The linear counterpart of model (3.5) is as follows:

$$\max \quad \sum_{p=1}^{n}(\sum_{i=1}^{m} s_i^{-p} + \sum_{r=2}^{s} s_r^{+p} + u^p + v^p)$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j^p x_{ij} + s_i^{-p} = \theta_p^* x_{ip},$$

$$i = 1,\ldots,m, \quad p = 1,\ldots,n,$$

$$\sum_{j=1}^{n} \lambda_j^p x_{ij} + s_i^{-p} = \theta_p^* x_{ip},$$

$$i = 1,\ldots,m, \quad p = 1,\ldots,n, \qquad (3.6)$$

$$\sum_{j=1}^{n} \lambda_j^p y_{rj} - (u^p - v^p) = y_{rp},$$

$$r = 1, \quad p = 1,\ldots,n,$$

$$\sum_{j=1}^{n} \lambda_j^1 y_{1j} + \ldots + \sum_{j=1}^{n} \lambda_j^n y_{1j} = c, \quad (a)$$

$$s_r^{+p} \geq 0, \quad u^p \geq 0, \quad v^p \geq 0,$$

$$r = 2,\ldots,s, \quad p = 1,\ldots,n,$$

$$\lambda_j^p \geq 0, \quad j = 1,\ldots,n, \quad p = 1,\ldots,n,$$

$$s_i^{-p} \geq 0, \quad i = 1,\ldots,m, \quad p = 1,\ldots,n.$$

In the following it is shown that when solving CCR model for these targets all of them are evaluated as efficient units. It is noteworthy to mention that in $T_c \subseteq R^2$ (one-input and one-output), the projection point in input orientation is always located onto the efficient frontier. You can see this situation in Figure 1.

Consider an input-oriented projection, a projection point is found by maximum proportional reduction in inputs without any change in output data. Thus, conventional target setting in $T_c \subseteq R^2$ is equivalent to the presented method. According to what has been discussed above in $T_c \subseteq R^2$ (one-input and one-output), the differences between this method and conventional one can not be seen. It is obvious that in higher dimensions where multiple outputs exist and positive output slacks can be found, the differences between this method and conventional one are more tangible. Therefore, to make matters more clearly in using this method, we turn to a schematic portrayal for an arbitrary T in Figure 2. In this figure solid lines and dotted lines indicate proportional changes in inputs and nonradial changes, respectively.

Figure1: Target setting (one-input and one-output)

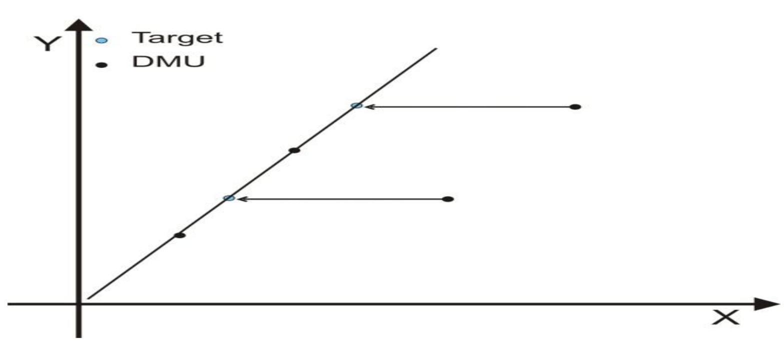
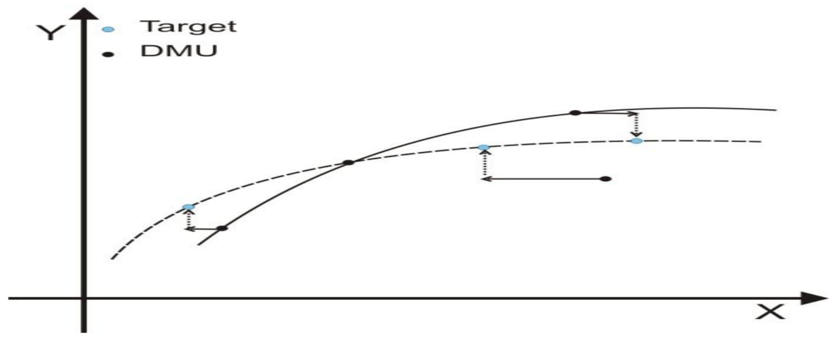


Figure2: Target setting (an arbitrary T)



Non radial changes can be a decrease or an increase in first element of output in order to set target. The dotted curve in this figure is the efficient frontier passing through all target units.

Theorem 1. There exist at least one unit from among n units that corresponding θ^* in model (3.4) is equal to one.

Proof. To prove this claim we will show that there exist a p and a DMU_j which corresponding inequality constraint in multiplier form of model (3.4) is binding. Consider model (3.4), since a solution with

$$\forall p, \lambda_p^p = 1; \quad \forall p, \theta_p = 1; \\ \forall j = 1, \dots, n, j \neq p, \lambda_j^p = 0, \tag{3.7}$$

Always exists, then the optimal value of objective function is less than or equal to n. Moreover, considering $\theta_p^* = 0$ for all p results in $\lambda_j^p = 0$, for all j and p which is in contradiction to constraint (a) because c is a positive constant. Thus, $0 < \sum_{p=1}^{n} \theta_p^* \leq n$. Now, consider the objective function of the corresponding multiplier form of model (?). Consequently, in accordance with the strong duality theorem the optimal value of objective function of enveloping and multiplier forms are equal. Thus the optimal value of objective function in corresponding multiplier model of model (?) is finite and positive that means;

$$0 < \sum_{p=1}^{n} \theta_p^* = \sum_{p=1}^{n} \sum_{r=1}^{s} u_r^{*p} \, y_{rp} + w^* c. \tag{3.8}$$

Hence, there exists at least one non zero term in objective function of multiplier form. Without loss of generality let exist k and l such that $u_l^{*k} y_{lk} > 0$, thus $u_l^{*k} \neq 0, y_{lk} \neq 0$. Now we will prove that there exist p and there exist DMU_j which corresponding constraint in optimal solution of multiplier form is binding. Consider the inequality constraints of multiplier form which is as following;

$$\sum_{r=1}^{s} u_r^p \, y_{rj} - \sum_{i=1}^{s} v_i^p \, x_{ij} + w y_{1j} \leq 0, \\ j = 1, \dots, n, p = 1, \dots, n. \tag{3.9}$$

By contradiction suppose that there exist no binding constraints in optimal solution hence;

$\forall p, \forall j; \sum_{r=1}^{s} u_r^{*p} y_{rj} - \sum_{i=1}^{s} v_i^{*p} x_{ij} + w^* y_{1j} < 0.$ (3.10)

Since $y_{lk} \neq 0$, if for all j, $y_{lj} = 0$ then let $\bar{\varepsilon_j} \in R^+$ and $\bar{u}_l^p = u_l^{*p} + \bar{\varepsilon}_j$. If for all j, $y_{lj} > 0$ then let $\bar{u}_l^p = u_l^{*p} + \varepsilon_j$. Now, by substituting $(\bar{u}, v^*, w^*)$ in (0.3) we will arrive at the following expression;

$$\varepsilon_j \leq \frac{\sum_{i=1}^{s} v_i^{*p} x_{ij} - \sum_{r=1}^{s} u_r^{*p} y_{rj} - w^* y_{1j}}{y_{lj}}$$

Now define; $\varepsilon = min\{\varepsilon_j, \forall j\}$. Therefore a feasible solution such as

$$\forall i, \forall p, \bar{v}_i^p = v_i^{*p}; \quad \bar{w} = w^*;$$
$$\forall r \neq l, \forall p, \bar{u}_r^p = u_r^{*p};$$
$$r = l, \forall p, \bar{u}_l^p = u_l^{*p} + \varepsilon,$$

is at hand for which there exist p and DMU_j such that;

$$\sum_{r=1}^{s} u_r^{*p} y_{rj} - \sum_{i=1}^{s} v_i^{*p} x_{ij} + w^* y_{1j} < \sum_{r=1}^{s} \bar{u}_r^{*p} y_{rj} - \sum_{i=1}^{s} \bar{v}_i^{*p} x_{ij} + \bar{w}^* y_{1j} \leq 0.$$

The important issue is that this feasible solution has an objective value which is greater than the optimum, thus leading to a contradiction and this completes the proof. Therefore, there exist a p and there exist DMU_j which corresponding constraint in optimal solution of multiplier form is binding. □

Theorem 2. Consider the set of target units, $DMU\ (\acute{x}, \acute{y})$ where $(\acute{x}, \acute{y}) = (\theta_k^* x_k - s^{-*k}, y_k + s^{+*k})$ for all k, each of them is efficient.

Proof. Let us assume that the proposition is false and we will arrive at contradiction. Assume that the target unit of DMU_p is not located on the efficient frontier thus there exists$(\hat{x}, \hat{y})$, $\hat{\lambda}$, $\hat{s}^-$ and $\hat{s}^+$ in corresponding PPS that;

$$\hat{x} = \sum_{j=1}^{n} \hat{\lambda}_j^k x_j + \hat{s}_i^{-k},$$
$$\hat{y} = \sum_{j=1}^{n} \hat{\lambda}_j^k y_j - \hat{s}_r^{+k}, \quad (3.11)$$
$$\hat{\lambda}^k \geq 0, \hat{s}^{-k} \geq 0, \hat{s}^{+k} \geq 0$$

Which dominates $(\acute{x}, \acute{y})$, i.e.; $\acute{x} \geq \hat{x}, \quad \acute{y} \leq \hat{y}$, and at least one equality is strict. Hence, there exists non negative vectors α and β that;

$$\sum_{i=1}^{m} \alpha_i + \sum_{r=1}^{s} \beta_r > 0, \quad (3.12)$$

Thus,

$$\acute{x}_i = \hat{x}_i + \alpha_i, \quad i = 1, \ldots, m,$$
$$\acute{y}_r = \hat{y}_r - \beta_r, \quad r = 1, \ldots, s. \quad (3.13)$$

According to model (3.5) in optimal solution we have the following expressions. For simplification of notation for all p, consider $s_1^{+*p} = u^{*p} - v^{*p}$.

$$\sum_{j=1}^{n} \lambda_j^{*k} x_{ij} + s_i^{-*k} = \theta_k^* x_{ik}, \quad \forall i$$
$$\sum_{j=1}^{n} \lambda_j^{*k} y_{rj} - s_r^{+*k} = y_{rk}, \quad \forall r,$$
$$\sum_{j=1}^{n} \lambda_j^{*p} x_{ij} + s_i^{-*p} = \theta_p^* x_{ip}, \forall i, \forall p \neq k,$$
$$\sum_{j=1}^{n} \lambda_j^{*p} y_{rj} - s_r^{+*p} = y_{rp}, \quad \forall r, \forall p \neq k.$$

Considering expressions (3.11) and (3.13) while assessing DMU_k define;

$\acute{x}_{ik} = \theta_k^* x_{ik} - s_i^{-*k} = \sum_{j=1}^{n} \hat{\lambda}_j x_{ij} + \hat{s}_i^- + \alpha_i, \quad i = 1,\ldots,m,$

$\acute{y}_{rk} = y_{rk} + s_r^{-*k} = \sum_{j=1}^{n} \hat{\lambda}_j y_{rj} - \hat{s}_r^+ - \beta_r, \quad r = 1,\ldots,s,$

Where;

$\theta_k^* x_{ik} = \sum_{j=1}^{n} \hat{\lambda}_j x_{ij} + \hat{s}_i^- + \alpha_i + s_i^{-*k}, \quad i = 1,\ldots,m,$

$y_{rk} = \sum_{j=1}^{n} \hat{\lambda}_j y_{rj} - \hat{s}_r^+ - \beta_r - s_r^{-*k}, \quad r = 1,\ldots,s.$

Therefore, while assessing $(\theta_k^* x_k, y_k)$ a feasible solution such as $(\hat{\lambda}, S^-, S^+)$ has been obtained in which;

$S_i^{-k} = \hat{s}_i{}^- + s_i^{-*k} + \alpha_i, \quad i = 1,\ldots,m,$

$S_r^{+k} = \hat{s}_r{}^+ + s_r^{+*k} + \beta_r, \quad r = 2,\ldots,s,$

$S_1^{+k} = \hat{s}_1{}^+ + s_1^{+*k} + \beta_1,$

$S_i^{-p} = s_i^{-*p}, \quad i = 1,\ldots,m, \quad \forall p \neq k$

$S_r^{+p} = s_r^{+*p}, \quad r = 1,\ldots,s, \quad \forall p \neq k$

It is worth mentioning that this is also a feasible solution for the second phase model. Thus, considering this solution we will acquire an objective value which is greater than the obtained optimum through solving the second phase. Therefore, we have arrived at a contradiction and proof is completed. By constructing a set of these new observations, all of these units, in a joint manner, are located onto a new efficient frontier that is, solving CCR model all of them will be assessed as efficient units. To make matters more concrete in use of this procedure we turn to a numerical example which involves 6 $DMUs$. The input-output data are tabulated in Table 1.

Solving model (3.5) resulted in the optimal strategy of each unit in which the first output can be decreased or increased or remained unchanged. By contribution of the constraint (a) in model (3.5), for all j any changes in y_{1j}, are implemented in a way that the resultant sum after changes equals the initial sum. An increase in the share of an individual will cause a decrease in the share of other ones and vice versa. As can be seen, any alterations in the first element of output through all units are under the influence of the alterations of other units. According to Table 2, the sum of s_1^{+*p} through all $DMUs$ is equal to zero. Considering what has been indicated in Table 2 for each inefficient unit by utilizing model (3.4) and (3.5) efficient targets can be found. The targets resulted from the presented two-stage procedure are gathered in Table3.

Table1. Inputs and Outputs

DMUp	I1	O1	O2	DMUp	I1	O1	O2
DMU1	59	31	28	DMU4	45	23	33
DMU2	28	22	33	DMU5	36	34	27
DMU3	57	30	26	DMU6	64	35	22

Table2. Optimal solutions of model (3.4) and (3.5)

DMUp	θ*p	S_1^{-*p}	S_1^{+*p}	S_2^{+*p}	DMUp	θ*p	S_1^{-*p}	S_1^{+*p}	S_2^{+*p}
DMU1	0.63	0	4.26	0	DMU4	0.62	0	-1	0
DMU2	1	0	0	0	DMU5	0.64	0	-16	0
DMU3	0.61	0	2.74	0	DMU6	0.74	0	10	13.74

Table3. Targets

DMUp	I1	O1	O2	DMUp	I1	O1	O2
DMU1	37.33	35.26	28	DMU4	28	22	33
DMU2	28	22	33	DMU5	22.91	18	27
DMU3	34.67	32.74	26	DMU6	47.65	45	35.74

When solving CCR model for the set of obtained targets all of them will be evaluated as efficient units. That means, these targets has been located on an efficient frontier which is passing through all of them.

Theorem 3. Share data remains share throughout the analysis.

Proof. Considering the target unit of $\mathrm{DMU_p}$ we will show that the sum of output element of this unit is equal to $\sum_{p=1}^{n} y_{1p}$By summing the constraints related to the first element of DMU_p over p we will have;

$$\sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_j^{*p}\, y_{1j}-\sum_{p=1}^{n}s_1^{+p}=\sum_{p=1}^{n}y_{1p}.$$

Taking into account constraint (a),

$$\sum\nolimits_{j=1}^{n}\lambda_j^{*1}\, y_{1j}+\cdots+\sum\nolimits_{j=1}^{n}\lambda_j^{*n}\, y_{1j}=$$

$\sum_{p=1}^{n} y_{1p}$ by comparing these two equalities we have; $\sum_{p=1}^{n} s_1^{*+p}=0$. Now, consider the first element of output of benchmark unit, by summing this output over p we have; $\sum_{p=1}^{n}(y_{1p}+s_1^{*+p})=\sum_{p=1}^{n} y_{1p}$. In accordance to what has been mentioned above since $\sum_{p=1}^{n} s_1^{*+p}=0$ thus share data remains share throughout the analysis. By summing of the first element of output through all units in Table 1 and 3, we can see that both of them are equal to the initial sum which is 175. One point to note is that the difference between this method and the conventional one due to existence of share data, are constraint (a) and the unrestricted slack. This constraint relates $DMUs$ to each other. Thus, the amount of the first element of output will be altered in such a way that the resulted sum after changes in this element remains equal to the initial sum. Due to existence of unrestricted non radial changes in the first element of output vector it should be noted that in different examples it is necessary to have multiple outputs with at least one regular element. Otherwise this method will not give interpretable results.

1. Application

In this section, an empirical example about the application of the proposed approach into Commercial banks is given. We consider eleven Commercial banks of Iran and the related input-output data are tabulated in Table 4. In summary, the input and output sets are as follows:

Inputs:

• Payable interest. / • Number of Personnel per hour. / • Non-performing loans.

Outputs:

• Total sum of the four main deposits. / • Other deposits. / • Loans granted. / • Received interest. / • Fee.

As it is mentioned above o_1 indicates the entire sum of four main deposits of each bank. This element indicates share data since the entire investment in banks is limited to these deposits. The sum of the invested money in all banks is different from those deposits which are not for investing. Thus, if a bank wants to increase its invested money, then the invested money of other banks are to be decreased. That means, any of these banks must convince people to invest their money in their bank. Thus the share of other banks from invested money will decrease. Therefore, any alterations in the share of the banks are related to each other.

Considering model (3.4) and (3.5) mentioned units in Table 4 are assessed in order to find suitable target.

Table4. Inputs and Outputs

DMUp	I1	I2	I3	O1	O2	O3	O4	O5
DMU1	4707.86	175.8	60801	1033890	42954	611224	31671.6	189.17
DMU2	32641.23	477.94	264991	5398005	966040	5090776	108826.2	2328.4
DMU3	24603.99	511.76	238510	5795565	871880	4839322	131011.6	2335.87
DMU4	9097.12	348.65	85897	2332104	815245	3284772	65056.46	2936.8
DMU5	34766.12	276.55	402614	4313779	539228	7878616	231066.5	2306.15
DMU6	41239.42	408.88	105778	6136069	298420	5115135	29197.01	1838.93
DMU7	24978.41	459.78	321776	4923925	1802130	4887652	123469.1	3580.4
DMU8	4902.54	254.34	110543	1097316	122046	1127011	12581.5	306.16
DMU9	2278.13	142.75	300084	555997	22165	168786	3672.26	137.19
DMU10	23642.26	736.26	58238	3736368	190077	1353879	23249.96	512.91
DMU11	8394.97	529.64	64750	1437663	60187	929473	20853.48	281.64

Table5. Optimal solution of model (3.5)

DMUp	S_1^{+*p}	DMUp	S_1^{+*p}	DMUp	S_1^{+*p}	DMUp	S_1^{+*p}
DMU1	85041.96	DMU4	0	DMU7	0	DMU10	-565552
DMU2	1779323	DMU5	0	DMU8	-297167	DMU11	-577289
DMU3	0	DMU6	0	DMU9	-424356		

As it is listed in Table 5, units 1 and 2 are to increase the share of their invested money and then units 8, 9, 10 and 11 are to decrease their share.

As can be seen in Table 5 the sum of s_1^{-*p} over p is equal to zero. Hence, shared data remains shared throughout the analysis. In accordance with the obtained optimal solution of model (3.4) and (3.5), target units can be found.

2. Conclusion

In this paper, while considering share date, we focused on finding suitable target units. Share data for each individual indicate the share of a unit from the whole amount. The basic idea behind this study is to make some changes in two-stage procedure in order to acquire target units, while the sum of specific element of output vector in benchmark units is still equal to the initial sum. In the presented procedure share data can be increased or decreased in such a way that finally after these changes the sum of resultant elements in benchmark units is still equal to the initial sum. The proposed model is in input orientation and the mentioned condition is on outputs. Due to existence of unrestricted non radial changes it should be mentioned that at least one regular element in output vector is required in order to gain interpretable results. Thus, further investigations of other concepts which are relevant to DEA and an analysis where all elements of output vector indicate share data, can be considered from this point of view.

References

[1] J. Aparicio, J.L. Ruiz, and I. Sirvent, Closest targets and minimum distance to the Pareto-efficient frontier in DEA, J Prod Anal. 28 (2007), pp.209–218.

[2] C. Baek, and J. Lee, The relevance of DEA benchmarking information and the Least-Distance Measure, Mathematical and Computer Modelling. 49 (2009), pp.265-275.

[3] R.D. Banker, A. Charnes, and W.W. Cooper, Some models for estimating technical and scale efficiencies in data envelopment analysis, Management Sience. 30 (1984), pp.1078-1092.

[4] A. Charnes, W.W. Cooper, and E. Rhodes, Measureing the efficiency of decision making units, European journal of operational reaserch. 2 (1978), pp.429-444.

[5] W.D. Cook, L.M. Seiford, and J. Zhu, Models for performance benchmarking: measuring the efficiect of e-business activities on banking performance, Omega. 32 (2004), pp.313–322.

[6] E. Gonzalez, and A. Alvarez, From efficiency measurement to efficiency improvemenT: The choice of a relevent benchmark, European journal of operational reaserch. 133 (2001), pp.512-520.

[7] J.L. Hougaard, and H. Keiding, Continuous benchmark selections. Operations Research Letters. 32 (2004), pp.94-98

[8] J.L. Hougaard, and M. Tvede, Benchmark selection: An axiomatic approach, European journal of operational reaserch. 137 (2002), pp.218–228

[9] C.N. Madu, and C. Kuei, Application of data envelop analysis in benchmarking, International Journal of Quality Science. 3 (1998), pp.320-327

[10] A. Post, and J. Spronk, An integrated benchmarking approach to distribution center performance using DEA modeling, Journal of Operations Management. 20 (1999), pp.19-32.

[11] T. Ross, and C. Droge, Performance benchmarking using interactive data envelopment analysis, European journal of operational reaserch. 115 (2002), pp.472 - 487.

[12] H. Seo, J. Choi, G. Park, and Y. Park, A framework for benchmarking service process using data envelopment analysis and decision tree, Expert Systems with Applications. 32 (2007), pp.432–440.

[13] A.J.C. Trappey, and Tzu-An Chiang, A DEA benchmarking methodology for project planning and management of new product development under decentralized profit-center business model, Advanced Engineering Informatics. 22 (2008), pp.438–444.

[14] J. Wu, L. Liang, and F. Yang, Achievement and benchmarking of countries at the Summer Olympics using cross efficiency evaluation method, European journal of operational reaserch. 197 (2009), pp.722-730.