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Fully Fuzzy Transportation Problem

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Abstract

Transportation problem is a linear programming which considers minimum cost for shipping a product from some origins to other destinations such as from factories to warehouse, or from a warehouse to supermarkets. To solve this problem simplex algorithmis utilized. In real projects costs and the value of supply and demands are fuzzy numbers and it is expected that optimal solutions for determining the value of commodities transported from a source to a destination be obtained as a fuzzy. So the first idea is to present the in the full fuzzy condition and then an algorithmwhich is of importance for solving such a problem. In this article, a new algorithm is suggested for solving fully fuzzy transportation problem. This algorithm transforms the fully fuzzy transportation problem into a triple-objective problem and then it utilizes a weighted method for solving multi-objective problems and solves the new problem using simplex transportation method. At the end, the suggested method is utilized for the real data.

Keywords: fully fuzzy transportation problem, triangular fuzzy numbers, linear multi objective programming.

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1- Introduction

The transportation problem has many application in solving problems of the real world. As an example, the transportation problem play a significant role in the logistics and management of the supply chain. Parametersof the transportation problem consist of amounts of cost, supply and demand. In the usual form of this problem the aforementioned parameters are fixed and definitive but in the real world the parameters are ambiguous and imprecise. After that the theory of fuzzy set was presented by Zadeh in 1965 [18], the problem of fuzzy was studied in relation to the real world. Available methods for solving fuzzy transportation problem present complex solution for fuzzy transportation. Shiang-Tai Liu and Chiang Kao [14], Chanas et al. [3], Chanas and Cochiti [2] have proposed methods for solving transportation problem. Nagoor Gani and Abdul Rezak [11] suggested a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Pandian et al. [13] proposed a method namely zero point method for finding a fuzzy optimal solution for a fuzzy transportation problem in which all parameters are trapezoidal fuzzy numbers. Amarpreet kaur [1] proposed a new method for solving fuzzy transportation problem by assuming that a decision maker is uncertain about precise values of the product. In many fuzzy decision problems, the data are represented in terms of fuzzy numbers. In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal. In order to change the fuzzy number into crisp form the ranking method is utilized. The ranking method was first proposed by Jain [8]. For ordering fuzzy quantities in [0, 1] Yager [15] suggested four indices which may be employed for this purpose. Ranking has a role in different parts of fuzzy optimization. For further study in this are a [4-5, 9-10] referencescan be utilized. The fully fuzzy transportation problem is a transportation problem in which parameters of cost, supply and demand are fuzzy values. The aim of fuzzy transportation is to find the leasttransportation cost of some commodities through anetwork when the supply and demand ofnodes and the capacity and cost of edges arefuzzy numbers. In this article we are to study fuzzy transportation problem when all parameters are triangular fuzzy numbers and transform it into a triple-objective problem and utilize a weighted method for solving the multiple-objective problem. At the end, using north- west corner method we are to calculate the feasible solution of the them and then continue to use the rest of the transportation simplex method in order to achieve the optimal solution.

2. Basic definitions

Definition 2-1.Assume that X is a nonempty set. Each fuzzy subset of $\tilde{A}$ from X is determined by the membership function of $\mu_{\tilde{A}}: X \to [0,1]$ in which for each $x \in X$ the value of $\mu_{\tilde{A}}(x)$ in the interval of $[0,1]$ shows the membership degree of x in $\tilde{A}$ [17].

Definition2-2. A fuzzy number is a fuzzy set like $u: R \to I = [0,1]$ which satisfies:

1. u is upper semi-continuous.

2. 0=u(x) outsidesomeinterval$[c, d]$.

3. There are real numbers of a, b exist such that $c \le a \le b \le d$ and

$u(x)$ ismonotonicincreasing on [c,d]

$u(x)$ ismonotonicdecreasing on [b,d]

- a≤x≤b 1 = u(x)

Definition 2-3.The parametric form of a fuzzy number u is a pair in functions $(\underline{u},\overline{u})$ in which functions of $\underline{u}(r)$ and $\overline{u}(r)$ for values of $0 \le r \le 1$ satisfy following conditions: [17, 18]

1. The u function is a bounded non-decreasing left semi-continuous function on the interval [0,1].

2. The $\overline{u}$ function is a bounded non-increasing left semi continuous function on the interval [0, 1].

3. $\underline{u}(r) \le \overline{u}(r)$ for the values $0 \le r \le 1$

Definition 2-4.Trapezoidal and triangular numbers: the more general forms of fuzzy numbers are the trapezoidal fuzzy numbers in the non-fuzzy distance [b,c] where (a-b) is the left defuzzier and (c-d) is the right defuzzier and the membership function of them is as follows:

$$\mu_{\tilde{u}}(x) = \begin{cases} \frac{x-a}{b-a} & if a \le x \le b \\ 1 & if b \le x \le c \\ \frac{d-x}{d-c} & if c \le x \le d \\ 0 & otherwise \end{cases}$$

In the parametric form,

$\underline{u}(r) = a + (b - c)r$,

$\overline{u}(r) = d + (c - d)r$

$0 \le r \le 1$.

In particular, $\tilde{u} = (a, b, c, d)$ can also signify a triangular fuzzy number $\tilde{u} = (a, b, d) = ((u)^l, (u)^c, (u)^u)$ if b=c

Point: the set of all triangular fuzzy numbers is denoted by TF(R).

2-1- Arithmetic operators on the triangular fuzzy numbers

In this section the arithmetic operations between two triangular fuzzy numbers defined on the real numbers R are presented.

Definition 2-5. [17, 18] assume that $\tilde{A}$= (a,α, β) and $\tilde{B}$=(b,γ, θ,) are the triangular fuzzy numbers and $x \in R$ is a non-zero scalar, then:

a) If $x \in R, x > 0$ then $\widetilde{Ax}$=$(xa\ , x\alpha, x\beta)$.

b) If $x \in R, x < 0$ then $\widetilde{Ax}$=$(xa\ ,- x\beta, -x\alpha)$.

c) $a+b\ , \alpha+\gamma\ , \beta+\theta)(\tilde{A} \oplus \tilde{B}=$

d) $\widetilde{\mathrm{A}} \ominus \widetilde{\mathrm{B}} = (a-\theta, \alpha-\gamma, \beta-b)$

e) $-\widetilde{\mathrm{A}} = -(a, \alpha, \beta) = (-\beta, -\alpha, -a)$

Definition 2-6.[17 and 18] The triangular fuzzy number (a, b, c) is the non-zero fuzzy number if ≥ 0 . The set of all nonnegative triangular fuzzy numbers is shown by $TF(R)^{+}$.

Definition 2-7. [19 and 20] two triangular fuzzy numbers $\widetilde{\mathrm{A}} = (a, b, c)$ and $\widetilde{\mathrm{B}} = (e, f, g)$ are equal if and only if$a$=e and$b$=$f$and$c$=g.

Definition 2-8. [17 and 18]. Assume that $\widetilde{\mathrm{A}} = (a, b, c)$ is a triangular fuzzy number and $\widetilde{\mathrm{B}} = (e, f, g)$ is a non-negative triangular fuzzy number, then:

$$\widetilde{\mathrm{A}\otimes}\widetilde{\mathrm{B}} = \begin{cases} (\mathrm{ae}, \mathrm{bf}, \mathrm{cg}) & \mathrm{a} \geq 0 \\ (\mathrm{ag}, \mathrm{bf}, \mathrm{cg}) & \mathrm{a} < 0, c \geq 0 \\ (\mathrm{ag}, \mathrm{bf}, \mathrm{ce}) & \mathrm{c} < 0 \end{cases}$$

Definition 2-9. Let $\tilde{u} = (a, b, c)$ and $\tilde{v} = (e, f, g)$ be tow arbitrary triangular fuzzy numbers. We say that $\tilde{u}$ is relatively less than $\tilde{v}$, which is by $\tilde{u} < \tilde{v}$, if and only if:

(i) $b < f$

(ii)$b = f and (c-a) > (g-e) or$

(iii) $b = c, (c-a) = (g-e) and (a+c) < (e+g)$.

Remark2.1: it is clear that
$b = c, (c-a) = (g-e) and\ (a+c) = (e+g)$ if and only if $\tilde{u} = \tilde{v}$.

Remark2.2: $\tilde{u} \leq \tilde{v}$. if and only if $\tilde{u} < \tilde{v}$. or $\tilde{u} = \tilde{v}$.

Remark3: A multi-objective problem is often solved by combining its multiple objectives into one single-objective scalar function. This approach is in general known as the weighted-sum or scalarization method. In more detail, the weighted-sum method minimizes a positively weighted convex sum of the objectives, that is,

$$\min \mathrm{WZ}(\mathrm{x}) = \sum\nolimits_{\mathrm{i}=1}^{\mathrm{p}} \mathrm{w_i z_i(x)}$$

$$\mathrm{s.t} \qquad \mathrm{x} \in \mathrm{X}$$

$$\mathrm{w} = (\mathrm{w_1}, \dots, \mathrm{w_k}) > 0$$

That represents a new optimization Problem with a unique objective function.

3- Fully fuzzy transportation problem

A Fully fuzzy transportation problem (FFTP) is a linear programming problem of a specific structure. If in transportation problem, all parameters and variables are fuzzy, we will have a fully fuzzy transportation problem as follows.

Suppose that there are m warehouses and $\tilde{a}_i$represents renders of warehouse i and n represents customer and $\tilde{b}_j$is the demand of customer j. $\tilde{c}_{ij}$is the cost of transporting one unit of product from warehouse i to the customer jand $\tilde{X}_{ij}$is the value of transported product from warehouse i to the customer j. The objective is to minimizethe cost of transporting a product from the warehouse to the customer.

$$Min\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\, \tilde{x}_{ij}$$

$$s.t \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i i = 1, \dots, m \qquad (1)$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \qquad j = 1, \dots, n$$

$$\tilde{x}_{ij} \geq 0 \qquad i = 1, \dots, mj = 1, \dots, n$$

With this assumption that the total value of demand equals to the total value of the supply, the transportation problem always has one feasible solution i.e.:

$$\tilde{X}_{ij} = \left((X_{ij})^l, (X_{ij})^c, (X_{ij})^u\right) = \frac{\tilde{a}_i \otimes \tilde{b}_j}{\tilde{d}}$$

$$= \frac{((a_i)^l, (a_i)^c, (a_i)^u) \otimes \left((b_j)^l, (b_j)^c, (b_j)^u\right)}{\tilde{d}}$$

$$i = 1,2, \dots, m \quad j = 1,2,, \dots, n$$

In which $\tilde{d} = \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ is a feasible solution. Since for each i and j $\tilde{a}_i > 0$ and $\tilde{b}_j > 0$, then $\tilde{X}_{ij} > 0$.

And we have

$$\sum_{i=1}^{m} \tilde{X}_{ij} = \sum_{i=1}^{m} \frac{\tilde{a}_i \otimes \tilde{b}_j}{\tilde{d}} = \frac{\tilde{b}_j}{d} \sum_{i=1}^{m} \tilde{a}_i = \frac{\tilde{b}_j \tilde{d}}{\tilde{d}} = \tilde{b}_j j$$
$$= 1, \dots, n$$

$$\sum_{j=1}^{n} \tilde{X}_{ij} = \sum_{j=1}^{n} \frac{\tilde{a}_i \otimes \tilde{b}_j}{\tilde{d}} = \frac{\tilde{a}_i}{\tilde{d}} \sum_{j=1}^{n} \tilde{b}_j = \frac{\tilde{a}_i \tilde{d}}{\tilde{d}} = \tilde{a}_i$$

Now we prove that:

$\tilde{X}_{ij} \leq Min\{\tilde{a}_i, \tilde{b}_j\}$

Proof: we have$\tilde{X}_{ij} = \frac{\tilde{a}_i \otimes \tilde{b}_j}{\tilde{d}}$

As $\tilde{b}_j < \sum_{j=1}^{n} \tilde{b}_j$, therefore $\frac{\tilde{b}_j}{\sum_{j=1}^{n} \tilde{b}_j} < 1$

So,

$$\tilde{X}_{ij} = \frac{\tilde{a}_i \otimes \tilde{b}_j}{\tilde{d}} = \tilde{a}_i(\tilde{b}_j/\tilde{d}) \leq \tilde{a}_i.1 = \tilde{a}_i$$

In the same way it can be shown that $\tilde{X}_{ij} \leq \tilde{b}_j$

Then,

$\tilde{X}_{ij} \leq Min\,\{\tilde{a}_i, \tilde{b}_j\}$

As the limited linear programing problem has a feasible solution, it has the optimal solution. Therefore we are looking for the optimal solution in the transportation problem.

4- Suggested Algorithm

In this section a new algorithm for solving fully fuzzy transportation problem is presented. The steps of proposed algorithm are given as follows:

Step1: with respect to definition (2-4) and (2-7), the problem (1) is converted to the following:

$$Min \quad \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left((C_{ij}X_{ij})^l, (C_{ij}X_{ij})^c, (C_{ij}X_{ij})^u \right)$$

$$s.t \quad \sum_{j=1}^{n} \left((X_{ij})^l, (X_{ij})^c, (X_{ij})^u \right) = ((a_i)^l, (a_i)^c, (a_i)^u) \quad i = 1, \dots, m \qquad (2)$$

$$\sum_{i=1}^{m} \left((X_{ij})^l, (X_{ij})^c, (X_{ij})^u \right) = ((b_i)^l, (b_i)^c, (b_i)^u) \quad j = 1, \dots, n$$

$$\tilde{X}_{ij} \geq 0 \qquad i = 1, \dots, m \qquad j = 1, \dots, n$$

Problem (2) may be rewritten as follows:

$$Min \quad \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left((C_{ij}X_{ij})^l, (C_{ij}X_{ij})^c, (C_{ij}X_{ij})^u \right)$$

$$s.t \sum_{j=1}^{n} (X_{ij})^l = (a_i)^l \ , \sum_{j=1}^{n} (X_{ij})^c = (a_i)^c \ , \sum_{j=1}^{n} (X_{ij})^u = (a_i)^u \ \ i = 1, .., m$$

$$\sum_{j=1}^{n} (X_{ij})^l = (b_j)^l \ , \sum_{j=1}^{n} (X_{ij})^c = (b_j)^c \ , \sum_{j=1}^{n} (X_{ij})^u = (b_j)^u \ \ j = 1, .., n$$

$$(X_{ij})^c - (X_{ij})^l \geq 0 \ , (X_{ij})^u - (X_{ij})^c \geq 0 \ , (X_{ij})^l \geq 0 \qquad (3)$$

$$i = 1, \dots, m \qquad j = 1, \dots, n$$

Step 2: Regarding to Definition 2.9 problem (3) is converted to the MOLP problem with three crisp objective functions as follows:

$$Min \qquad (C_{ij}X_{ij})^c$$

$$Max \quad (C_{ij}X_{ij})^u - (C_{ij}X_{ij})^l$$

$$Min \quad (C_{ij}X_{ij})^u + (C_{ij}X_{ij})^l \qquad (4)$$

$$s.t \sum_{j=1}^{n} (X_{ij})^l = (a_i)^l \ , \sum_{j=1}^{n} (X_{ij})^c = (a_i)^c \ , \sum_{j=1}^{n} (X_{ij})^u = (a_i)^u \ \ i = 1, .., m$$

$$\sum_{j=1}^{n} (X_{ij})^l = (b_j)^l \ , \sum_{j=1}^{n} (X_{ij})^c = (b_j)^c \ , \sum_{j=1}^{n} (X_{ij})^u = (b_j)^u \ \ j = 1, .., n$$

$$(X_{ij})^c - (X_{ij})^l \geq 0$$

$$\& (X_{ij})^u - (X_{ij})^c \geq 0 \ \& \ (X_{ij})^l \geq 0$$

Step 3: Regarding to Remark3 a specific weight in assigned to each objective of problem (3) and from the sum of weighted objective an objective goal is obtained

$$Min \quad 0/5 \, (C_{ij}X_{ij})^c - 0/2(\, (C_{ij}X_{ij})^u - (C_{ij}X_{ij})^l \,) + 0/3(\, (C_{ij}X_{ij})^u + (C_{ij}X_{ij})^l) \qquad (5)$$

$$s.t \sum_{j=1}^{n} (X_{ij})^l = (a_i)^l \ , \sum_{j=1}^{n} (X_{ij})^c = (a_i)^c \ , \sum_{j=1}^{n} (X_{ij})^u = (a_i)^u$$

$$i = 1, .., m$$

$$\sum_{j=1}^{n} (X_{ij})^l = (b_j)^l \ , \sum_{j=1}^{n} (X_{ij})^c = (b_j)^c \ , \sum_{j=1}^{n} (X_{ij})^u = (b_j)^u \ \ j = 1, .., n$$

$$(X_{ij})^c - (X_{ij})^l \geq 0$$

$$\& (X_{ij})^u - (X_{ij})^c \geq 0 \ \& \ (X_{ij})^l \geq 0$$

Step 4: for each of the new cost coefficient and its related supply and demand a table of transportation is drawn and each of them is solved using simplex method in transportation problem, i.e.:

1- For starting U, we find a feasible basic solution utilizing north- west corner method.

2- We obtain values of $(z_{ij})^{\mathrm{l}} - (\mathrm{c_{ij}})^{\mathrm{l}}, (z_{ij})^{\mathrm{c}} - (\mathrm{c_{ij}})^{\mathrm{c}}, (z_{ij})^{\mathrm{u}} - (\mathrm{c_{ij}})^{\mathrm{u}}$ for each of non-basic variable in each table. We stop here or select the input column.

3- We determine the output column.

4- We achieve a new basic feasible solution and repeat the stage 2.

Now we are to solve two problems using the proposed algorithm.

Practical Example

In this section, with regard to fuzzy parameters two examples are presented and then solved using the proposed algorithm.

Example 1. Consider transportation problem presented in Table 1.

Through putting above values in the weighted function of model (5), new cost coefficients are achieved (table2).

We convert table (2) into tables (3), (4) and (5) as follows

After completing stage (1) of step 4, we come up with basic feasible solutions (tables 7,8,9)

Table 1. Data of the transportation problem

Supply	O_2	O_1	
(150,201,246)	(22,31,34)	(15,19,29)	D_1
(50,99,154)	(30,39,54)	(8,10,12)	D_2
	(100,150,200)	(100,150,200)	Demand

Table 2. New cost coefficients

Supply($\tilde{a}_i$)	O_2	O_1	
(150,201,246)	(11,15.5,3.4)	(7.5,9.5,2.9)	D_1
(50,99,154)	(15,19.5,5.4)	(4,5,1.2)	D_2
	(100,150,200)	(100,150,200)	Demand($\tilde{b}_j$)

Table 3. Lower bound of costs

${a_i}^{\mathrm{l}}$	${O_2}^{\mathrm{l}}$	${O_1}^{\mathrm{l}}$	
150	11	7.5	${D_1}^{\mathrm{l}}$
50	15	4	${D_2}^{\mathrm{l}}$
	100	100	${b_j}^{\mathrm{l}}$

Table 4. Middle bound of costs

$a_i{}^{c}$	$O_2{}^{c}$	$O_1{}^{c}$	
201	15.5	9.5	$D_1{}^{c}$
99	19.5	5	$D_2{}^{c}$
	150	150	$b_j{}^{c}$

Table 5. Upper bound of costs

$a_i{}^{u}$	$O_2{}^{u}$	$O_1{}^{u}$	
246	3.4	2.9	$D_1{}^{u}$
154	5.4	1.2	$D_2{}^{u}$
	200	200	$b_j{}^{u}$

Table 6. The middle bound of basic feasible solutions

$a_i{}^{l}$	$O_2{}^{l}$	$O_1{}^{l}$	
150	50	100	$D_1{}^{l}$
50	50		$D_2{}^{l}$
	100	100	$b_j{}^{l}$

Table 7. The lower bound of basic feasible solutions

$a_i{}^{c}$	$O_2{}^{c}$	$O_1{}^{c}$	
201	51	150	$D_1{}^{c}$
99	99		$D_2{}^{c}$
	150	150	$b_j{}^{c}$

Table 8. The upper bound of basic feasible solutions

$a_i{}^{u}$	$O_2{}^{u}$	$O_1{}^{u}$	
246	46	200	$D_1{}^{u}$
154	154		$D_2{}^{u}$
	200	200	$b_j{}^{u}$

With performing stage (2) of step 4, we will have:

$z_{21}{}^{l} - c_{21}{}^{l} = 7.5 - 11 + 15 - 4 = 7.5$

So, $x_{21}{}^{l}$ is inserted into the base and $\Delta = \min\{50,100\} = 50$, therefore:

$x_{11}{}^{l} = 100 - 50 = 50$

$x_{12}{}^{l} = 50 + 50 = 100$

$x_{21}{}^{l} = 0 + 50 = 50$

$x_{22}{}^{l} = 50 - 50 = 0$

Then, $x_{22}{}^{l}$ is removed from the base and new obtained values are optimized because

$z_{22}{}^{l} - c_{22}{}^{l} = 11 - 7.5 + 4 - 15 = -7.5$

Through conducting these stages on other tables the following optimal values are achieved:

$x_{11}{}^{l} = 50 \quad x_{11}{}^{c} = 51 \quad x_{11}{}^{u} = 46$

$x_{12}{}^{l} = 100 \quad x_{12}{}^{c} = 150 \quad x_{12}{}^{u} = 200$

$x_{21}{}^{l} = 50 \quad x_{21}{}^{c} = 99 \quad x_{21}{}^{u} = 154$

$x_{22}{}^{l} = 0 \quad x_{22}{}^{c} = 0 \quad x_{22}{}^{u} = 0$

Example (2). Dali Company is the leading producer of soft drinks and low-temperature foods in Taiwan. Currently, Dali plans to develop the South-East Asian market and broaden the visibility of Dali products in the Chinese market. Notably, following the entry of Taiwan to the World Trade Organization, Dali plans to seek strategic alliance with prominent international companies. To meet demand of its clients, Dali utilizes four distribution centers C_1, C_2, C_3, C_4 and three factories F_1, F_2, F_3. The transportation costs for each way are paid by Dali at the end. For example, the available supply of F_1 (7.2, 8, 8.8) is thousand dozen bottles. The forecast demand of distribution center C_1 (6.2, 7, 7.8) is thousand dozen bottles, and the transportation cost per dozen bottles from F_1 to $C_1$is (\$8,\$10,\$10.8). Due to transportation costs being a major expense, the management of Dali is initiating a study to reduce these costs as much as possible. (**Table 9)** [19]

The coefficients of cost are summarized in three tables **10, 11, 12.**

Utilizing the northwest corner method, the basic feasible solutions **Table 13, 14, 15.**

Table 9. Data of example 2

supply	C_4	C_3	C_2	C_1	
(7.2,8,8.8)	(18.8,20,22)	(8,10,10.6)	(20.4,22,24)	(8,10,10.8)	F_1
(12,14,16)	(6,8,8.8)	(10,12,13)	(18.2,20,22)	(14,15,16)	F_2
(10.2,12,13.8)	(14,15,16)	(7.8,10,10.8)	(9.6,12,13)	(18,20,21)	F_3

Table 10. The lower bound of new costs

$a_i{}^{l}$	$C_4{}^{l}$	$C_3{}^{l}$	$C_2{}^{l}$	$C_1{}^{l}$	
7.2	9.4	4	10.2	4	$F_1{}^{l}$
12	3	5	9.1	7	$F_2{}^{l}$
10.2	7	3.9	4.8	9.2	$F_3{}^{l}$
	7.8	6.5	8.9	6.2	$b_j{}^{l}$

Table 11. The middle bound of new costs

$a_i{}^{c}$	$C_4{}^{c}$	$C_3{}^{c}$	$C_2{}^{c}$	$C_1{}^{c}$	
8	10	5	11	5	$F_1{}^{c}$
14	4	6	10	7.5	$F_2{}^{c}$
12	7.5	5	6	10	$F_3{}^{c}$
	9	8	10	7	$b_j{}^{c}$

Table 12. The upper bound of new costs

$a_i{}^{u}$	$C_4{}^{u}$	$C_3{}^{u}$	$C_2{}^{u}$	$C_1{}^{u}$	
8.8	6.6	3.18	7.2	3.24	$F_1{}^{u}$
16	2.64	3.9	6.6	4.8	$F_2{}^{u}$
13.8	4.8	3.24	3.9	6.3	$F_3{}^{u}$
	10.2	9.5	11.1	7.8	$b_j{}^{u}$

Table 13. The lower bound of basic feasible solutions

$a_i{}^{l}$	$C_4{}^{l}$	$C_3{}^{l}$	$C_2{}^{l}$	$C_1{}^{l}$	
7.2			1	6.2	$F_1{}^{l}$
12		4.1	7.9		$F_2{}^{l}$
2.10	7.8	2.4			$F_3{}^{l}$
	7.8	6.5	8.9	6.2	$b_j{}^{l}$

Table 14. The middle bound of basic feasible solutions

a_i^{c}	C_4^{c}	C_3^{c}	C_2^{c}	C_1^{c}	
8			1	7	F_1^{c}
14		5	9		F_2^{c}
12	9	3			F_3^{c}
	9	8	10	7	b_j^{c}

Table 15. The upper bound of basic feasible solutions

a_i^{u}	C_4^{u}	C_3^{u}	C_2^{u}	C_1^{u}	
8.8			1	7.8	F_1^{u}
16		5.9	10.1		F_2^{u}
13.8	10.2	3.6			F_3^{u}
	10.2	9.5	11.1	7.8	b_j^{u}

Calculating $\tilde{Z}_{ij} - \tilde{C}_{ij}$ and determining the input variable:

$Z_{13}^{l} - C_{13}^{l} = 5 - 9.1 + 10.2 - 4 = 1.9$

$Z_{14}^{l} - C_{14}^{l} = 7 - 3.9 + 5 - 9.1 + 10.2 - 9.4 = -0.2$

$Z_{21}^{l} - C_{21}^{l} = 4 - 10.2 + 9.1 - 7 = -4.1$

$Z_{24}^{l} - C_{24}^{l} = 7 - 3.9 + 5 - 3 = 4.1$

$Z_{31}^{l} - C_{31}^{l} = 3.9 - 5 + 9.1 - 10.2 + 4 - 9.2 = -7.4$

$Z_{32}^{l} - C_{32}^{l} = 9.1 - 5 + 3.9 - 4.8 = 3.2$

So, x_{24}^{l} is entered into the base.

$Z_{13}^{c} - C_{13}^{c} = 11 - 9 + 6 - 5 = 3$

$Z_{14}^{c} - C_{14}^{c} = 7.5 - 5 + 6 - 10 + 11 - 10 = -0.5$

$Z_{21}^{c} - C_{21}^{c} = 5 - 11 + 10 - 7.5 = -3.5$

$Z_{24}^{c} - C_{24}^{c} = 6 - 5 + 7.5 - 4 = 4.5$

$Z_{31}^{c} - C_{31}^{c} = 5 - 11 + 10 - 6 + 5 - 10 = -7$

$Z_{32}^{c} - C_{32}^{c} = 10 - 6 + 5 - 6 = 3$

So, x_{24}^{c} is entered into the base.

$Z_{13}^{u} - C_{13}^{u} = 3.9 - 6.6 + 7.2 - 3.18 = 1.32$

$Z_{14}^{u} - C_{14}^{u} = 4.8 - 3.24 + 3.9 - 6.6 + 7.2 - 6.6 = -0.54$

$Z_{21}^{u} - C_{21}^{u} = 3.24 - 7.2 + 6.6 - 4.8 = -2.16$

$Z_{24}^{u} - C_{24}^{u} = 4.8 - 3.24 + 3.9 - 2.64 = 2.72$

$Z_{31}^{u} - C_{31}^{u} = 3.24 - 7.2 + 6.6 - 3.9 + 3.24 - 6.3 = -4.32$

$Z_{32}^{u} - C_{32}^{u} = 6.6 - 3.9 + 3.24 - 3.9 = 2.04$

So, x_{24}^{u} is entered into the base.

Determining the output variable:

$\Delta_1^{l} = min\{7.8, 4.1\} = 4.1$

$x_{23}^{l} = 4.1 - 4.1 = 0$

$x_{24}^{l} = 0 + 4.1 = 4.1$

$x_{33}{}^{l} = 2.4 + 4.1 = 6.5$

$x_{34}{}^{l} = 7.8 - 4.1 = 3.7$

$x_{23}{}^{l}$ is removed from the base.

$\Delta_1{}^{c} = min\{9,4\} = 4$

$x_{23}{}^{c} = 5 - 5 = 0$

$x_{24}{}^{c} = 0 + 5 = 5$

$x_{33}{}^{c} = 3 + 5 = 8$

$x_{34}{}^{c} = 9 - 5 = 4$

$x_{23}{}^{c}$ is removed from the base.

$\Delta_1{}^{u} = min\{5.9,10.2\} = 5.9$

$x_{23}{}^{u} = 5.9 - 5.9 = 0$

$x_{24}{}^{u} = 0 + 5.9 = 5.9$

$x_{33}{}^{u} = 3.6 + 5.9 = 9.5$

$x_{34}{}^{u} = 10.2 - 5.9 = 4.3$

$x_{23}{}^{u}$ is removed from the base.

We continue this method up to the time tables are optimized, i.e. when the condition of $\tilde{Z}_{ij} - \tilde{C}_{ij} \leq 0$ is hold. The optimal solution is as follows:

$$\begin{cases}
\tilde{x}_{11}{}^{*}=\left((x_{11})^{l},(x_{11})^{c},(x_{11})^{u}\right)=(6.2,7,7.8)\\
\tilde{x}_{12}{}^{*}=\left((x_{12})^{l},(x_{12})^{c},(x_{12})^{u}\right)=(0,0,0)\\
\tilde{x}_{13}{}^{*}=\left((x_{13})^{l},(x_{13})^{c},(x_{13})^{u}\right)=(1,1,1)\\
\tilde{x}_{14}{}^{*}=\left((x_{14})^{l},(x_{14})^{c},(x_{14})^{u}\right)=(0,0,0)\\
\tilde{x}_{21}{}^{*}=\left((x_{21})^{l},(x_{21})^{c},(x_{21})^{u}\right)=(0,0,0)\\
\tilde{x}_{22}{}^{*}=\left((x_{22})^{l},(x_{22})^{c},(x_{22})^{u}\right)=(0,0,0)\\
\tilde{x}_{23}{}^{*}=\left((x_{23})^{l},(x_{23})^{c},(x_{23})^{u}\right)=(4.2,5,5.8)\\
\tilde{x}_{24}{}^{*}=\left((x_{24})^{l},(x_{24})^{c},(x_{24})^{u}\right)=(7.8,9,10.2)\\
\tilde{x}_{31}{}^{*}=\left((x_{31})^{l},(x_{31})^{c},(x_{31})^{u}\right)=(0,0,0)\\
\tilde{x}_{32}{}^{*}=\left((x_{32})^{l},(x_{32})^{c},(x_{32})^{u}\right)=(8.9,10,11.1)\\
\tilde{x}_{33}{}^{*}=\left((x_{33})^{l},(x_{33})^{c},(x_{33})^{u}\right)=(1.3,2,2.7)\\
\tilde{x}_{34}{}^{*}=\left((x_{34})^{l},(x_{34})^{c},(x_{34})^{u}\right)=(0,0,0)
\end{cases}$$

6. Conclusion

Studying fully fuzzy transportation problems due to its close connection with human life, is considered to be of great importance. The preliminary of solving the transportation problem has to do withfully fuzzy linear programming. Therefore, in this articlethrough using a proposed algorithm infully fuzzy transportation problem and getting ideas from methods of solving multi-objective linear programming problems afuzzy optimal solution was obtained. The proposed algorithm becomes a multi-objective problem which is significant for being used in interactive methods for making any comment by related managers and achieving the logical solutions. For further studies, solving fully fuzzy transportation problemthrough taking into account the minimal cost and time is recommended.

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